Name: ______________ Math 256A Quiz 4 Due Thursday 9/26

1. For each ring R, find the number of irreducible components of $\operatorname{Spec} R$.
   (a) $R_n = \mathbb{C}[x,y]/I_n$, where $I_n = (x^n - y^n)$, for $n = 1, 2, 3, \ldots$.
   (b) $R_c = \mathbb{C}[x,y]/I_c$, where $I_c = (x^2 + y^2 + 1, xy - c)$, for $c \in \mathbb{C}$.
   (c) $R_\lambda = \mathbb{C}[x,y,z]/I_\lambda$, where $I_\lambda = (x^2 + y^2 + z^2, \lambda)$, for $\lambda = ax + by + cz$ with $a, b, c \in \mathbb{C}$.

2. (a) Suppose $X = \operatorname{Spec} R$ is an affine scheme and $f \in R \simeq \mathcal{O}_X(X)$ is a global function. Show that the open subspace $D(f) \subset X$ equipped with the restricted structure sheaf is again an affine scheme.
   (b) Let $\mathbb{A}^2 = \operatorname{Spec} \mathbb{C}[x,y]$ be the affine plane, and $0 \in \mathbb{A}^2$ the closed point given by the prime ideal $(x,y) \subset \mathbb{C}[x,y]$. Show that the open subset $\mathbb{A}^2 \setminus 0$ equipped with the restricted structure sheaf is not an affine scheme.

3. Give an example of a map of affine schemes $f : X = \operatorname{Spec} R \to \operatorname{Spec} S = Y$ such that its image $f(X) \subset Y$ is not locally closed (= open in its closure).