Name: ______________ Math 256A Quiz 3 Due Thursday 9/19

1. Let $X = \mathbb{C}$ be the complex line with its classical topology. Let $f : \mathbb{C} \to \mathbb{C}$, $f(x) = x^2$ be the square map.
   (a) Describe the pullback $f^*\mathbb{C}_X$ of the constant sheaf.
   (b) Describe the pushforward $f_*\mathbb{C}_X$ of the constant sheaf, in particular, its indecomposable summands.
   (c) Can you find a sheaf of $\mathbb{C}$-vector spaces $\mathcal{F}$ on X such that $f_*\mathcal{F} \simeq \mathbb{C}_X$?

2. Let $\pi : Y = \mathbb{C}^2 - \{0\} \to \mathbb{CP}^1 = X$, $\pi(x, y) = [x, y]$ be the natural map from the punctured complex plane to the complex projective line, both equipped with the classical topology. Let $\mathcal{F}$ be the sheaf of holomorphic sections of π. So to $U \subset X$ open, $\mathcal{F}$ assigns

$$\mathcal{F}(U) = \{s = (f, g) : U \to \mathbb{C}^2 \setminus \{0\}, \pi \circ s = id, \text{ with } f, g \text{ holomorphic}\}$$

   (a) Calculate the sections of $\mathcal{F}$ over the listed open sets

$$A_+ = \{[x, y]|x \neq 0\} \qquad A_- = \{[x, y]|y \neq 0\} \qquad A_+ \cap A_- = \{[x, y]|x, y \neq 0\}$$

   (b) Use the previous calculation to calculate the global sections of $\mathcal{F}$.

3. Let X be a topological space, and X_{disc} the same set but with the discrete topology.
   (a) Show the identity map of sets is continuous $f : X_{disc} \to X$.
   (b) For any sheaf $\mathcal{F}$ of abelian groups on X, calculate the sheaf $f_*f^*\mathcal{F}$ on X.
   (c) Describe the canonical map (unit of adjunction) $\mathcal{F} \to f_*f^*\mathcal{F}$, and show it is injective.
   (d) Show $f_*f^*\mathcal{F}$ is flabby. (A sheaf of abelian groups is flabby if its restriction maps are surjective.)

   Remark for homological algebraists: iterating the construction of part (c) gives a canonical flabby resolution of any sheaf of abelian groups called the *Godement resolution*.