1. For each polynomial below, first describe its affine zero locus, then (minimally) homogenize the polynomial, describe the corresponding projective zero locus, in particular its points on the line at infinity.
   (a) $p(x, y) = x^3 - y$
   (b) $p(x, y) = x^3 - y^2$
   (c) $p(x, y) = x^3 + x^2 - y^2$

2. Let $\mathrm{SL}(2, \mathbb{C})$ be the set of 2×2 complex matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   with determinant satisfying $ad - bc = 1$.
   (a) Show that the entries of the product and inverse of elements of $\mathrm{SL}(2, \mathbb{C})$ are given by polynomials in the entries of the given elements.
   (b) Consider the natural action of $\mathrm{SL}(2, \mathbb{C})$ on the projective space $\mathbb{P}^1_{\mathbb{C}}$ of lines through the origin in $\mathbb{A}^2_{\mathbb{C}}$ induced by the linear action on $\mathbb{A}^2_{\mathbb{C}}$. Given a line

$$\ell = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \in \mathbb{P}^1_{\mathbb{C}}$$

   of slope x_1/x_0, write a formula for the slope of the line obtained by applying

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathrm{SL}(2, \mathbb{C})$$

   (c) Given three distinct lines $\ell_0, \ell_1, \ell_\infty \in \mathbb{P}^1_{\mathbb{C}}$, show there is an element $A \in \mathrm{SL}(2, \mathbb{C})$ that takes $\ell_0, \ell_1, \ell_\infty \in \mathbb{P}^1_{\mathbb{C}}$ to lines of the respective slopes $0, 1, \infty$. Is the element A unique?

3. Prove that any function $f : \mathbb{P}^n_{\mathbb{C}} \to \mathbb{C}$ whose restriction to each affine chart $A_i = \{x_i \neq 0\}$, for $i = 0, \ldots, n$, is a polynomial must in fact be constant.