- 1. (10 points) State whether each assertion is always true (T) or sometimes false (F).
 - (a) (1 point) <u>T</u> For any object x of any category \mathcal{C} , the hom-set $\operatorname{Hom}_{\mathcal{C}}(x,x)$ is non-empty.
 - (b) (1 point) <u>F</u> If a functor is an equivalence, it is a bijection on objects.
 - (c) (1 point) $\underline{\mathbf{F}}$ The Yoneda functor $Y: \mathcal{C} \to \operatorname{Fun}(\mathcal{C}^{op}, \operatorname{Set}), Y(x) = \operatorname{Hom}_{\mathcal{C}}(x, -)$ is an equivalence.
 - (d) (1 point) <u>F</u> All objects of a groupoid are isomorphic to each other.
 - (e) (1 point) $\underline{\mathbf{F}}$ The forgetful functor $F: \mathrm{Grp} \to \mathrm{Set}$ is full.
 - (f) (1 point) $\underline{\mathbf{T}}$ The forgetful functor $F: \text{Grp} \to \text{Set}$ is faithful.
 - (g) (1 point) $\underline{\mathbf{T}}$ The forgetful functor $F: AbGrp \to Grp$ is full.
 - (h) (1 point) $\underline{\mathbf{T}}$ The forgetful functor $F: \mathrm{AbGrp} \to \mathrm{Grp}$ is faithful.
 - (i) (1 point) $\underline{\mathbf{T}}$ The forgetful functor $F: \mathrm{AbGrp} \to \mathrm{Grp}$ preserves products.
 - (j) (1 point) $\underline{\mathbf{F}}$ The forgetful functor $F: \mathrm{AbGrp} \to \mathrm{Grp}$ preserves coproducts.
- 2. (10 points) Let \mathcal{R} be the category with objects real numbers $r \in \mathbb{R}$ and hom-sets

$$\operatorname{Hom}_{\mathcal{R}}(r,s) = \left\{ \begin{array}{cc} \{\bullet\} & r \leq s \\ \emptyset & r > s \end{array} \right.$$

- (a) (2 points) i. State (Yes or No) if the product (in the sense of category theory!) of the objects 2 and 3 exists in \mathcal{R} .
 - i. ____Yes

ii. If yes, calculate it.

- ii. _____2
- (b) (2 points) i. State (Yes or No) if the coproduct of the objects 2 and 3 exists in \mathcal{R} .
 - i. ____Yes

ii. If yes, calculate it.

- ii 2
- (c) (2 points) i. State (Yes or No) if the limit (in the sense of category theory!) of the diagram

$$\cdots \longrightarrow 1/4 \longrightarrow 1/3 \longrightarrow 1/2 \longrightarrow 1$$

exists in \mathcal{R} .

i. ____Yes

ii. If yes, calculate it.

- ii. ____0
- (d) (2 points) i. State (Yes or No) if the colimit of the diagram

$$\cdots \longrightarrow 1/4 \longrightarrow 1/3 \longrightarrow 1/2 \longrightarrow 1$$

exists in \mathcal{R} .

i. <u>Yes</u>

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ii. If yes, calculate it.

ii. _____1

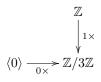
(e) (2 points) i. State (Yes or No) if \mathcal{R} is equivalent to the opposite category \mathcal{R}^{op} .

i. Yes

ii. If yes, define an equivalence $F: \mathcal{R} \to \mathcal{R}^{op}$ by specifying its map on objects.

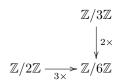
ii. F(r) = -r

- 3. (8 points) For each diagram of abelian groups, calculate its limit.
 - (a) (4 points)



(a) $\underline{\mathbb{Z}}$

(b) (4 points)



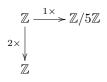
(b) _______(0)_____

- 4. (8 points) For each diagram of abelian groups, calculate its colimit.
 - (a) (4 points)



(a) $\mathbb{Z}/4\mathbb{Z}$

(b) (4 points)



(b) $\mathbb{Z}/10\mathbb{Z}$

- 5. (12 points) Let CRing be the category of commutative rings.
 - (a) (2 points) What is the initial object of CRing?

(a) <u>Z</u>

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	(b) (2 points) What is the terminal object of CRing?		
		(b)	(0)
	Recall for commutative rings R, S , their coproduct is the tensor product $R \otimes S$.		
	(c) (4 points) Calculate the tensor product $\mathbb{Q} \otimes \mathbb{Z}/n\mathbb{Z}$ as a function of n .		
		(c)	(0)
	(d) (4 points) Calculate the tensor product $\mathbb{Z}/m\mathbb{Z}\otimes\mathbb{Z}/n\mathbb{Z}$ as a function of m, n .		
		(d) <u>Z</u>	$/\gcd(m,n)\mathbb{Z}$
6.	(12 points) For a category \mathcal{C} , consider the identity functor $\mathrm{Id}_{\mathcal{C}}: \mathcal{C} \to \mathcal{C}$, i.e. the function object $x \in \mathcal{C}$ to itself and each morphism $f: x \to y$ to itself.	nctor th	at takes each
	For each listed category \mathcal{C} , calculate the group $\operatorname{Aut}(\operatorname{Id}_{\mathcal{C}})$ of automorphisms of $\operatorname{Id}_{\mathcal{C}}$ vertible natural transformations $\phi: \operatorname{Id}_{\mathcal{C}} \to \operatorname{Id}_{\mathcal{C}}$.	, i.e. the	e group of in-
	(a) (4 points) $C = \text{Vect}_k$ the category of k-vector spaces over a field k.		
		(a)	$k^{ imes}$
	(b) (4 points) $C = \text{FinSet}$ the category of finite sets.		
		(b)	$\langle 1 \rangle$
	(c) (4 points) $C = BH$ the classifying category of a group H , i.e. the category w hom-set $\operatorname{Hom}_{BH}(\bullet, \bullet) = H$ and composition given by mutiplication in H .	ith one	object • with
		(c)	Z(H)