The introduction of numbers as coordinates ... is an act of violence.

"Throug 3.3

Thus 9/26 Midterm I in Lecture Meeting

The G.3 through 5.4.2

The return of Rn1

Welcome to Lecture 9 Parks and Coords
(Equivalently, 3') $H$ is nonempty.

3) Contains zero vector: $\emptyset \in H$.

2) Closed under scalar: for any number $c \in \mathbb{C}$, $cH = \mathbb{C}H$. 

$H + \bar{H} \subseteq \text{closed under adj: } \bar{H}, \bar{H} \subseteq H$. 

Def. A subspace $H \subseteq V$ in a subset $V$ is a subspace of ambient vector space. 

How do real spaces often arise in nature?
1) \( \mathbb{R}^3 \) is a vector space, check axioms.

- \( \overline{0} \) is origin. (check axioms;)

- For all \( \mathbf{v} \) in \( \mathbb{R}^3 \), \( \mathbb{R}^3 \) spanned by \( \mathbf{v} \) itself.

- \( \text{Span} \{ \mathbf{v} \} \) spanned.

- \( \text{Span} \{ \mathbf{v}, \mathbf{v} \} \) spanned.

Ex: \( \mathbb{R}^3 \) (check axioms, point H itself is a vector space.)

- \( \mathbb{R}^3 \) is spanned. (check axioms;)

- \( \mathbb{R}^3 \) is itself spanned.
Here are some subspaces

\[ \{ x \} \quad \{ -x \} \quad \{ (x) - f(-x) \} \quad \{ 0 \} \]

1. \( H = \{ f \mid f \text{ is odd} \} \)
2. \( H = \{ f \mid f \text{ is even} \} \)
3. \( H = \{ f \mid f \text{ is differentiable} \} \)
4. \( H = \{ f \mid f \text{ is \text{ not} differentiable} \} \)
5. \( V = \{ f \mid f : \mathbb{R} \to \mathbb{R} \} \)
Image \( T \) is a \( W \) \( \cap \) \( \text{Null}(T) \) \( \subset \) \( \text{im} \) linear transformation. Vector spaces in turn are subsets of homog. "possible values for this system of equations". "sols of homog."
\[ \lambda \geq \mu, \mu \in H \cap H', H \subseteq H' \]

Subspaces are not the same.

2. \( H', H'' \subseteq \Lambda \)

Not a subspace; \( \emptyset \notin H \)

For some \( \bar{b} \notin \bar{b} \)

\( H = \) solution set of \( Ax = \bar{b} \)

\( A \) is a matrix.

1) (Caution: Non-examples are out there!)

\( \checkmark \)
But $H \cup H$ is only a subspace if one contains the other.

$H \cup H$ is a subspace.

Otherwise not closed under addition.
What should we do when we encounter an abstract vector space? Pray it's $\mathbb{R}^n$!

If not, try to relate it to $\mathbb{R}^n$.

Def: A basis $\beta$ of a vector space $V$ is a list of vectors $v_1, \ldots, v_k$ such that

1) $v_1, \ldots, v_k$ spans $V$ ("not too small")
2) $v_1, \ldots, v_k$ lin indep ("not too big")
Think of Goldilocks: bases are just right.
Main Point: \( B = \{\bar{\mathbf{x}}, \ldots, \mathbf{x}_k\} \) is a lin combo

\( \mathbf{v} = a_1 \mathbf{x}_1 + \ldots + a_k \mathbf{x}_k \) (*)

for some \( a_1, \ldots, a_k \)

(2) \( \iff \) (coeffs \( a_1, \ldots, a_k \) are unique)
We're done.

\[ \begin{align*}
\alpha = \alpha_1, \ldots, \alpha_r = \alpha_r \\
\text{and } a_1 - a_1 = 0, \ldots, a_r - a_r = 0
\end{align*} \]

\( (\text{SH5}) \quad (\text{SH7}) \quad \overline{0} \quad (x) - (xx) - (x) \)

Some other \( q_i \), \( i \leq r \)

Why? Suppose also \( \overline{v} = a_1 y_1 + \ldots + a_r y_r \)

\((***)\)
Exercise: Check $T^k$ is in $\text{lin tran}$. 

\[
\vec{v} = a_1 \vec{v}_1 + \ldots + a_k \vec{v}_k
\]

where

\[
T(\vec{v}) = \begin{bmatrix} a_1 \\
\vdots \\
a_k \end{bmatrix} = P \begin{bmatrix} p_1 \\
\vdots \\
p_k \end{bmatrix}(\vec{v})
\]

\[
\text{lin transf } T^k : V \to W
\]

(coord map with respect to $P$ is)

Def $V$ vec sp, $P = \{\vec{v}_1, \ldots, \vec{v}_k\}$ vec basis
is also do nothing!

\[
\exists x \in A \forall y \in B \quad \forall z \in C
\]

Observe \( T \) is not invertible!
\[ \vec{v} = A \vec{e}_1 + \cdots + A \vec{e}_n \]

\[ \text{Coordinates} = \text{Coordinates} \]

\[ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ \cdots \\ 0 \\ 0 \\ 1 \end{bmatrix} = 1 \]

Post std. basis

Example: \( v = \mathbb{R}^n \)
\[ p(x) = a_0 + a_1 x + \cdots + a_n x^n \]

\[ p(x) = a_0 x + a_1 x^2 + \cdots + a_n x^n \]

\[ \underbrace{\text{roots}} = \underbrace{\text{coeffs}} \]

1. \[ p_0(x) = 1 \]
2. \[ p_1(x) = x \]
3. \[ p_2(x) = x^2 \]
4. \[ p_3(x) = x^3 \]

Test std basis

\[ \forall \, x \in \mathbb{R}, \ p(x) = a_0 + a_1 x + \cdots + a_n x^n \]

If \( n \geq \deg \) of \( a \), then \( p \in \mathbb{R}[x] \)
\[ f(x) = \sum_{n=0}^{\infty} \left( a_n \cos(nx) + b_n \sin(nx) \right) \]

Fourier series: \( \cos\left(\frac{2\pi}{a}\right), \sin\left(\frac{2\pi}{a}\right) \)

Is there useful basis ...

\[ \begin{align*}
  V &= \{ \text{continuous functions} \} \\
  V^* &= \text{Counting measures} \\
  * &\text{Counting orthogonal} \\
\end{align*} \]
\[ \bar{y} = 3 \cdot \lfloor \frac{y}{10} \rfloor + 1 \cdot \lfloor y - \frac{y}{10} \rfloor \]

**Solution**

\[ \mathbf{P}_{\text{std}}(\bar{y}) = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \]

**Problem:**

What are the coordinates of \( \bar{y} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \) in \( \mathbb{P}^2 \)?

**Exercise:** \( \bar{y} = \mathbb{R}^2 \)
For \( P \), we need to solve
\[ V = \mathbb{R}^3 \]