Muhammad Ali!

You pay for your room here on earth.

Service to others is the rent.

Eff: Give through 9.1 NS5

Wed office hours are back! 12-2pm, 891 Ems.

Lett's get ready to rumble!

Variation of Parameters

Lecture 22: Underdetermined Coeffs
Roots: \( r_1 = 2, r_2 = -1 \)

Factors:
\[
(x - 2)(x + 1) = 0
\]

Aux. eqn:
\[
r^2 - r - 2 = 0
\]
\[
y - y\frac{r}{2} - 2y = 0
\]

Homo eqn:
\[
y\left(\frac{r}{2} - 2\right) = 0
\]

Focus for today: Finding a soln to the two order IVP (homogeneous and non-homogeneous)
\[ y' = \hat{a}e - \hat{t}e + \hat{a}t + \hat{a}t^2 \]

\[ \hat{y}' = 2\hat{a} + 2\hat{a}t + \hat{a}t^2 \]

\[ \hat{y}' = \hat{a}t + \hat{a}t^2 \]

To find \( a \): so try \( \hat{y}' = \alpha t^2 \)

Under coefs f = \( e \) is root of aux eqn

Back to nonhomog eqn

\( a = e, \; y = e^t \); \( y = e^t_1 + e^t_2 \)

Basis of solns to homog eqn
So \( yp = \frac{3}{1 + 2t} \).

Since \( f = e^{2t} \), conclude that \( a = \frac{1}{3} \).

Substituting back, \( yp - \frac{1}{4}p - 2yp = 3ae^{2t} \).
\[ v^2_y + v^2_z = \left(\frac{1}{2} v^2_y \right) + \left(\frac{1}{2} v^2_z \right) = v^2_y + v^2_z \]

To find \( v^2_y \) and \( v^2_z \):

Try \( v^2_y = \frac{1}{2} v^2 + v^2 = \frac{1}{2} v^2 + v^2 - t \)

Now try Variation of Parameters
Let's assume \( y_1', y_2' \) to avoid later appearance of 2nd deriv
\[ (x) \]
so \( y' = y_1 + y_2 \)

Given (*) 1, we have \( y_p = y_1' + y_2' \)

imply
\[ (**) \]
Subst back into eqn ...

\[
\begin{align*}
  &y_1 + y_2 = f \\
  &y_1' + y_2' + b(y_1' + y_2' - y_p) = f \\
  &y_1' + y_2' = \frac{f + y_p}{b}
\end{align*}
\]
"lin syst" = lin syst for each t

\[
\begin{bmatrix}
  y_1 & y_2 \\
  0 & y_1 \\
  y_1 & y_2
\end{bmatrix}
\]

\[(**)

\[(*)\]

\text{Now use } (x_1) + (**) \text{ to solve for } y_1, y_2

\text{Key if only lst devices appeal!}

\text{Observe } (x) + (**) \text{ arc "lin syst" for } y, y'.
Note: Coeff matrix is invertible for all $h_1, h_2$.

Since $w = a + e + h_1 + h_2$, $w$ is invertible for all $h_1, h_2$.

Since $y_1^2 = a + e + h_1 + h_2$, $y_2$.

Note: Coeff matrix is invertible for all $h_1, h_2$.
\[
\begin{align*}
W &= y_1y_2' \\
M &= \begin{bmatrix}
y_1 & y_2' \\
y_2' & -y_1 \\
\end{bmatrix}
\frac{M}{I} = \\
&= \begin{bmatrix}
y_1 & y_2' \\
y_2' & -y_1 \\
\end{bmatrix}
\begin{bmatrix}
y_1' \\
y_2' \\
\end{bmatrix} = \\
&= \begin{bmatrix}
y_1 \times t \\
y_2 \times t \\
\end{bmatrix}
\end{align*}
\]

It against any. vec.

Matrix and mult.

So to solve "Lin syst" invert coeff.
Good news: Find \( v_{22} \) by integrating.

Bad news: \( \int \frac{\text{d}y}{y_1 - y_2} \)

Conclude

\[ v_{22} = \frac{1}{y_2 - y_1} \]
\[ v_2 = \frac{e^{4t}}{-3e^t} \]
\[
\frac{v_1}{3} = \frac{e^t}{-3e^t} + C
\]
So find:
\[ v_1 = \int \frac{1}{3e^t} \, dt = \frac{1}{3} + C \]
\[ v = \int -3e^t \, dt = 1 + C \]

\[ W = 2e^{2t} \cdot e^{2t} - e^{-t} - e^{-t} (2e^{2t}) \]
\[ y_1 = e^{2t}, \quad y_2 = e^{-t} \]
\[ f = e^{2t} \]
To find simple soln, take $c_1 = c_2 = 0$. Then

\[ y_h = \frac{3}{1}te^{-t} + \frac{b}{1}e^{-t} + \frac{c_2}{1}e^{-t} + \frac{-1}{1}e^{-t} + \frac{2}{1}e^{-t} + \frac{2}{1}e^{-t} + \frac{1}{1}e^{-t} + \frac{1}{1}e^{-t} = y_p \]

Grand conclusion
Undertakers: \[ \sum (\ldots ) \hspace{1cm} \]

\[ y'' + y = \cos \theta(t) \]

Find a soln to

On to Round 2

Round 1

1. Winners: Undertakers

2. Knockout

by knockout
to use real basis

\[ u_1 = \cos(t) \]
\[ u_2 = \sin(t) \]

Since

\[ f = \sec(t) \]
\[ \csc(t) \]

for homogenous

basis of solns

Time:

\[ y_1 = e^t \]
\[ y_2 = e^{-t} \]
\[ r_1 = 1 \]
\[ r_2 = -1 \]
\[ y = c_1 e^t + c_2 e^{-t} \]

Aux:

\[ y = x_1 + y \]
\[ x_1 = 0 \]
\[ y = 0 \]
\[ x_1 = 1 \]
\[ y = 0 \]
so \( v' = \int_{t}^{t + \Delta t} \frac{\dot{v}'}{\sqrt{\dot{v}^2 + \dot{y}^2 - y'^2}} \) \( \Delta t \)

\( v' = \frac{\dot{v}'}{\sqrt{\dot{v}^2 + \dot{y}^2 - y'^2}} \) \( y' = \sqrt{y''^2} \)

Simplify route as before gives
\[ v_1 = \int -t + c \ dt = -\frac{\sin(t)}{\sin(t)} + c \]

\[ v_2 = \int \sqrt{(\cot(t))^4 + 4} \]

\[ v_1 = \frac{\sin(t)}{\sin(t)} = 1 \]

\[ f = \frac{1}{\sqrt{1 + \frac{\sin^2(t)}{\sin^2(t)}}} = \sin(t) \]

Back to specific forms: \( n_1 = \cos(t), n_2 = \sin(t) \)
\[ y_p = v_1 u_1 + v_2 u_2 \]

\[ = (-t + c_1) \cos(t) + (\ln |\sin(t)| + c_2) \sin(t) \]

Set \( c_1 = c_2 = 0 \) to find a single soln

\[ y_p = -t \cos(t) + \ln |\sin(t)| \sin(t) \]

Round 2 to Vaw of Params!
Wait a second! Have you taught us anything new?!

Claim All nth order lin ODE can be written as a system of 1st order lin ODE!
Set: $x_1 = 1 \quad \text{and} \quad x_2 = y$.

Think of it as an independent fn.

$Ex_{1, 2} = \{0 \} + by_1 + cy = f$.
\[ \begin{align*}
    \lambda(t) &= \bar{q} + x \left[ \begin{array}{cc}
        q & -2 \\
        1 & 0
    \end{array} \right] = x \\
    \begin{bmatrix}
        f+h-
        \\
        hq-
    \end{bmatrix}
    &=
    \begin{bmatrix}
        \bar{q} \\
        h
    \end{bmatrix}
    \\
    &= \begin{bmatrix}
        \bar{x} \\
        \bar{x}
    \end{bmatrix}
    = x
\end{align*} \]

Then