Guests bring good luck

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Next week: Back to normal.

Friday: No classes = No guts!

Lecture: Andrew Hanlon

Homogeneous Order Linear 2nd Nonhomogeneous 17 Lecture 21
\[
R = \frac{2}{4+\sqrt{16-20}} = 4 + \sqrt{4+\sqrt{2}}
\]
Auxiliary equation
Soln:

1) Find general solution
2) Find a particular solution (IVP)

\[y(0) = 3, \quad y'(0) = -4\]

\[y'' - 4y' + 5y = 0\]
\[ e^{(2+i)t} = e^2 e^{it} = e^2 \left( \cos t + i \sin t \right) \]

\[ e^{2+i} = \cos \theta + i \sin \theta \]

Euler's formula

But, these are complex-valued functions.

Actually, \[ z = 2 \pm i \]

Still a basis of solutions to \[ e^{(2+i)t} + z \] solutions
\[ y(t) = 2e^{2t} (\cos(t) + \sin(t)) + y(0) = \frac{3}{2} y'(0) + 3 \]

Real General Solution is:

Conclusion:
Summary of homogeneous case:

\[ y(t) = e^{2t} (3 \cos t - 10 \sin t) \]

Particular Solution:

\[ c_1 = \frac{1}{2}, \quad c_2 = 2, \quad c_3 = -10 \]

1 Always a 2-dimensional space

\[ y + 6y' + 2y = 0 \]
\[ \{ e^t \cos(kt), e^t \sin(kt) \} \text{ - basis of } \mathbb{R}^2 \]

2) One repeated root \( r \)

\[ \{ e^{rt}, te^{rt} \} \text{ - basis} \]

3) Complex conjugate roots \( \alpha \pm \beta i \)

\[ \{ e^{\alpha t}, e^{\beta t} \} \text{ - basis} \]

1) \( L \neq 2 \), distinct real roots

\[ p + q + r + s = 0 \]

Auxiliary equation

\[ y'' + 6y' + cy = 0 \]
Moreover:

**Wronskian lemma:** For a basis of solutions \( \{y_1, y_2\} \)

\[
\begin{bmatrix}
y_1(0) & y_2(0) \\
y_1'(0) & y_2'(0)
\end{bmatrix}
\]

is always invertible.

That is,

\[
\begin{bmatrix}
y_1(0) & y_2(0) & A \\
y_1'(0) & y_2'(0) & B
\end{bmatrix}
\]

always consistent.

Can solve

\[
y(0) = A \\
y'(0) = B
\]
Homogeneous: \( f(y) = 0 \)

Non-homogeneous: \( f \)

Non-linear differentiable function:

\[ f : \mathbb{R} \to \mathbb{R} \]

Let \( V \) be the vector space of infinitely differentiable functions on \( T : V \to V \) where \( V \) is the vector space of linear transformations.

\[ T : \frac{d^2y}{dt^2} + b \frac{dy}{dt} + cy + f = 0 \]
where \( \text{vec} \ \text{Nul}(A) \).

We can always write \( \overline{x} = \overline{x_0} + \overline{y} \) and go to a particular solution (one can)

Recall: \( A \overline{x} = \overline{b} \)

Row reduce? Not an option.

How did we solve \( A \overline{x} = \overline{b} \)?

How to solve: \( T(y) = 0 \).
Figure out how to do 1:

1) Find one solution
   \( \overline{x} = \frac{9}{2} \)

2) Solve \( \overline{Ax} = \overline{0} \)

3) \( \overline{x} + \overline{y} \) where \( \overline{y} \) are solutions

When put \( \overline{T} = \overline{A} \overline{x} + \overline{b} + \overline{c} \)

From part 2.
And, some products:

\[ \sqrt{\sec(x) \cdot \tan(x)}, 2 \sin(4t) \].

Also, consider:

\[ f(t) = t, \ \text{et}, \ \cos(4t), \ \sin(4t) \].

Consider any consider nice functions:

\[ \text{Answer: Make a good guess} \]

\[ y'' + by' + cy = f(t) \]

How are we going to find one solution?
Find \( y(t) = A^2 t^2 + A_1 t + A_0 \)

The span of these polynomials:

\[ + \]

\[ T: T \leftarrow 1 \leftarrow 2 \]

\[ \frac{d^2}{dt^2} + \frac{3}{2} + 2 \]

\[ T = \frac{4 t^2}{2} + \frac{3}{2} \frac{t^4}{4} + 2 \]

**Exer:** Find a soln to \( y'' + 3y' + 2y = t^2 \)

**Note:** Roots of auxiliary equation \(-1 \pm 2\)
\[ A_0 = -1 + \sqrt{\frac{1}{2}} \]
\[ 2A_0 = 1 + \sqrt{\frac{1}{2}} = \frac{3}{2} \]

\[ A_1 = -3 \]
\[ A_2 = \frac{\sqrt{5}}{2} \]

\[
\begin{bmatrix}
3 & A_0 & 0 & 0 \\
0 & 3 & 2 & 0 \\
3 & 0 & 2 & 0 \\
0 & 0 & 2 & 0
\end{bmatrix}
\] 

\[
\begin{bmatrix}
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -3 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] 

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] 

\[
\begin{bmatrix}
A_0 \ A_1 \ A_2 \ A_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 & 3 & 2 & 0 \\
0 & 3 & 2 & 0 \\
2 & 0 & 3 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\] 

Want to solve (graph $3 + 1/3 	imes 3 + 1$)?
\[ y(t) = e^{-t} + e^{2t} + \frac{1}{2} t^2 + \frac{1}{2} t - \frac{3}{2} t^2 - 2t + 2 \]

\[ \text{A Particular Solution} \]

\[ \begin{align*}
    y'' + 3y' + 2y &= f(t) \\
    y'' + \frac{7}{2} t^2 + 2t^2 + 2t + 2 &= f(t)
\end{align*} \]
Find coefficients by solving

$$y(t) = A_0 + A t + A t^2 + \cdots + A t^n$$

with solution of the form

$$y = b y' + c y''$$

for some constants $b$ and $c$.

Theorem: We can always solve (27) for a particular solution.
Exer. Find a soln. to

\[ y'' + 3y' + 2y = e^{3t} \quad \text{and} \quad y(0) = 2, \; y'(0) = 0 \]

A particular sol. is

\[ y = \frac{e^{3t}}{2} + 2e^{3t} \]
\[ \frac{d}{dt} (te^{-2t}) = -2e^{-2t} - 2e^{-2t} + hte^{-2t} \]

\[ \frac{d}{dt} (te^{-2t} + (-2t)e^{-2t}) = hte^{-2t} \]

\[ T(te^{-2t}) = 0 \]

Guess \( e^{-2t} \)

\[ 3y + 2y = e^{-2t} \]

Exercise: Find a solution to
\[ y(t) = -te^{-2t} + 1 \]
Solution