Welcome Back!

Lecture 2: Solving lin systs cont.

This week: Wed HW 1 due
Office Hrs 12-2pm
891 Evans

Fri Quiz through §1.4

"It is our attitude at the beginning of a difficult task which, more than anything else, will affect its successful outcome." William James

Hey, Prof - what did you mean by:

<u>Key idea of lin eqns</u> Suppose $\underline{\underline{b}} = 0$

Then sums and scales of solns are again solns.

<u>Understand for ex</u> $x_1 + 2x_2 + 3x_3 = 0$

<u>Soln</u> Try $x_3 = 0$ so $x_1 + 2x_2 = 0$

Set $x_2 = s$, $s \in \mathbb{R}$, so $x_1 = -2s$

Some solns are $(-2s, s, 0)$

Try also $x_2 = 0$ so $x_1 + 3x_3 = 0$

Set $x_3 = t$, $t \in \mathbb{R}$, so $x_1 = -3t$

Some solns are $(-3t, 0, t)$

Consider two solns when $s = t = 1$

$(-2, 1, 0)$ $(-3, 0, 1)$

Any soln is of form

$$s \cdot (-2, 1, 0) + t(-3, 0, 1), \quad s, t \in \mathbb{R}$$
$$= (-2s - 3t, s, t)$$

<u>To solve lin systs</u>

Strategy change to *equiv* but *simpler* lin syst

using row ops

(R1) Add mult of any row to any other

(R2) Exchange rows

(R3) Scale row by nonzero number

Observe: row ops change lin systs to equiv lin systs.

<u>Goal</u> Put lin syst in ~~[illegible]~~ Row Echelon Form (REF)

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \blacksquare & * & * & \cdots & * & \cdots & & * \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \blacksquare & * & \cdots & * \\ \vdots & \vdots & & \vdots & \vdots & 0 & \cdots & 0 & 0 & \blacksquare & * \cdots & * \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots & 0 & \cdots 0 & \blacksquare \\ \vdots & \vdots & & & & \vdots & & \vdots & \vdots & \vdots & & 0 \\ & & & & & & & & & & & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$\blacksquare$ = pivot / leading term $\neq 0$

$*$ = any number

<u>Exer</u> For what c, d is this in REF?

$$\begin{bmatrix} \boxed{2} & 0 & 1 & -1 & 5 \\ 0 & \boxed{3} & 0 & 3 & 1 \\ 0 & d & 0 & 0 & c \end{bmatrix}$$

$d \to 0$

<u>$c \neq 0$</u>
then pivot
so 3 pivots

<u>$c = 0$</u>
not pivot
so 2 pivots

$\boxed{\begin{array}{l} d = 0 \\ \text{any } c \end{array}}$

Even simpler Reduced REF (RREF)

$$
\begin{bmatrix}
0 & 0 & \boxed{1} & * & * & 0 & * & * & * & 0 \\
\vdots & \vdots & 0 & 0 & 0 & \boxed{1} & * & * & * & 0 \\
 & & \vdots & & & 0 & 0 & \cdots & 0 & \boxed{1} \\
 & & \vdots & & & \vdots & & & & 0 \\
 & & \vdots & & & \vdots & & & & \vdots \\
0 & 0 & 0 & \cdots & & 0 & & \cdots & & 0
\end{bmatrix}
$$

Pivots = 1

0 above pivots

Exer For what c,d is this in RREF?

$$\begin{bmatrix} c & 0 & 1 \\ 0 & d & 1 \end{bmatrix}$$

Cases

- $\underline{c=1}$
  - $\underline{d=1}$ RREF!
  - $\underline{d=0}$ not in <u>R</u>REF
  - $\underline{d\neq 0,1}$ not in <u>R</u>REF
- $\underline{c=0}$ not in REF so not in RREF
- $\underline{c\neq 0,1}$ not in <u>R</u>REF

Easy to solve lin systs in (R)REF

<u>Ex</u>

$$\left[\begin{array}{cccc:c} \boxed{1} & 2 & 0 & 2 & 3 \\ 0 & 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

pivot cols

x_1, x_3 pivot vars

x_2, x_4 free vars

Set free vars to be any numbers

$x_2 = s \;,\; x_4 = t \;,\; s, t \in \mathbb{R}$

Solve for pivot vars

$x_3 = -t$ (2nd equation)

$x_1 = 3 - 2s - 2t$ (1st equation)

Soln set = $\{ (3-2s-2t, s, -t, t) \}_{s,t \in \mathbb{R}}$

plane

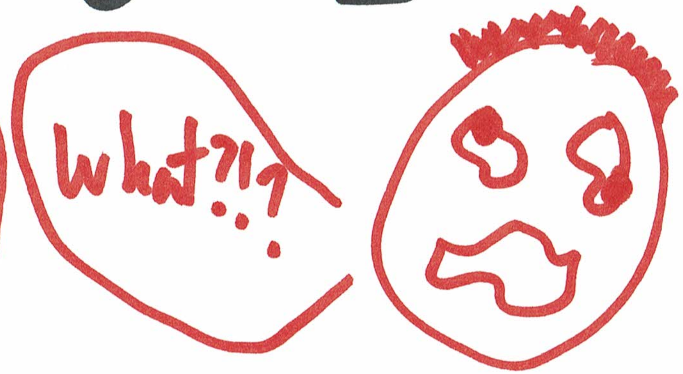
Caution!

Ex
$$\left[\begin{array}{ccc:c} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \end{array}\right]$$

last col has pivot!

row of this pivot: $0 = 1$

Lin syst inconsistent (has no solns)
$\Updownarrow$
Pivot in last column



Thm / Algorithm Given any (augmented) matrix, it's possible to put it in REF by row ops. In fact, it's possible to put in RREF uniquely

Ex

$$\begin{bmatrix} 0 & 0 & 0 & -1 & -2 \\ 0 & \boxed{2} & -2 & -2 & -1 \\ 0 & 2 & -2 & 1 & 5 \end{bmatrix}$$

Step 1 Find leftmost nonzero col.

Step 2 Find nonzero entry of this col

Want this to be first pivot so apply

$$\xrightarrow{(R2)} \begin{bmatrix} 0 & \boxed{2} & -2 & -2 & -1 \\ 0 & 0 & 0 & -1 & -2 \\ 0 & 2 & -2 & 1 & 5 \end{bmatrix}$$

Step 3 Create 0s below pivot so apply

$$\xrightarrow{(R1)} \begin{bmatrix} 0 & \boxed{2} & -2 & -2 & -1 \\ 0 & 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 & 6 \end{bmatrix}$$

<u>Step</u> 4 Repeat Steps 1-3 for submatrix

$$\begin{bmatrix} 0 & \boxed{2} & -2 & -2 & -1 \\ 0 & 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 3 & 6 \end{bmatrix}$$

...

<u>Voila!</u> REF
$$\begin{bmatrix} 0 & \boxed{2} & -2 & -2 & -1 \\ 0 & 0 & 0 & \boxed{-1} & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

<u>Step</u> 5 (for RREF)

Use (R3) to make pivots = 1

Use (R1) to create 0s above pivots

$$\xrightarrow{(R3)} \begin{bmatrix} 0 & \underline{1} & -1 & -1 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R1} \begin{bmatrix} 0 & \underline{1} & -1 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{RREF!}$$

<u>Next topic</u> lin syst with emphasis on <u>cols</u> not rows.

<u>Def</u> An n-vector is an ordered list of n numbers

$$\underline{u} = \begin{bmatrix} a_1 \\ \vdots \\ \vdots \\ a_n \end{bmatrix}$$

$\underline{u}$ n-vector

a_i numbers

3 Equiv ways to present lin syst

1) Eqns
$$2x_1 - 3x_2 + x_3 = 2$$
$$x_1 - x_2 - x_3 = 0$$

2) Aug matrix
$$\left[\begin{array}{ccc:c} 2 & -3 & 1 & 2 \\ 1 & -1 & -1 & 0 \end{array}\right]$$

3) Vector eqn
$$x_1\begin{bmatrix}2\\1\end{bmatrix} + x_2\begin{bmatrix}-3\\-1\end{bmatrix} + x_3\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}2\\0\end{bmatrix}$$