You back the right one.
A librarian can bring answers.
Google can bring you back 100,000

Fri, June 5th 4:30 PM
12-2 PM
891 Evans

Wednesday Hours

Eigenvalues & Eigenvectors

Applications of

Lecture 13
Primary rank according to appearance of keywords. School rank according to being most useful. Problem rank of pages that will Google's PageRank Algorithm.
New idea: rank pages by importance.

1) Popularity: how many pages link to your page.
2) Authority: how important the pages linking to your page are.

Page importance is a function of:

- Popularity
- Authority
I = \# \text{ of components} \downarrow \text{Vectors} \quad \underline{X} = [x_i]_{i \in I}

\text{Important: if website } i \in I \text{ we write } x_i = \text{importance of website } i \in I.

\text{We'll write } X_i = \text{importance of } \mathcal{I} \text{ set of webpages } \mathcal{I} = \text{Internet}.

\text{Some notation: let's write } I = \text{internet}.
Example mini-inference $I=\{1,2,3,4\}$

- Arrows are weighted by \# of arrows on a page.
What vector should the importance weights be organized into a matrix?
Importance $x_i$ scaled by weight
all other pages of the
importance $x_i$ is sum over

E-value $e_i$ - vector $e_i$ for $i=1$

$A \bar{x} = \bar{x}$

Simplicities
Find e-vector for $A$ with e-value $k = 1$.

\[ E_1 = N^\perp (A - I) \]

e-vectors are $x \neq 0 \in E$.

\[ \overline{x} \cdot x = \begin{bmatrix} 6 \\ 9 \\ 4 \\ 12 \end{bmatrix} \]

... solution $x =$
More theoretical application

Suppose \( V \rightarrow W \) linear transform

\[ \dim V = n \]
\[ \dim W = m \]

Recall choice of bases \( B_V, B_W \)

Provide matrix

A = \[ T_{B_V} \mid T_{B_W} \]
More delicate situation
\[ \dim V = n \]

Suppose \[ V \xrightarrow{T} V \]

How nice can we make matrix of \[ T \]

by choosing single basis of \[ V \]

if \[ \beta \] is a basis of \[ V \]

A = T_p T_p \vdots T_p
Then suppose $\beta = \{v_1, \ldots, v_n\}$ is a basis of e-vectors for $T$

$$Tv_i = \lambda_i v_i$$

Then matrix $A$ of $T$ wrt $\beta$

is diagonal

$$A = \begin{bmatrix}
\lambda_1 & 0 & \cdots & 0 \\
0 & \ddots & \cdots & 0 \\
\cdots & \cdots & \ddots & 0 \\
0 & \cdots & 0 & \lambda_n
\end{bmatrix}$$

"As nice as possible"
Now calc. i'th col.:

\[ T^i \vec{v} = T^i \vec{v} = (\nu_i \vec{v}) = \begin{bmatrix} T^i \vec{v} & \vdots & T^i \vec{v} \end{bmatrix} \]

Matrix \( A \) is given by

\[ A = \begin{bmatrix} T^1 \vec{v} & \cdots & T^n \vec{v} \end{bmatrix} \]
\[ A = \begin{bmatrix}
\ddots & & & & \\
& 0 & \ddots & & \\
& & 0 & 0 & \\
& & & \ddots & 0 \\
& & & & 0
\end{bmatrix} \]

So conclude
A \in \text{lin } V = n

\text{ is diagonalizable if there is a basis } \mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\} \text{ of } V \text{ wrt } \mathcal{B}

\text{ such that matrix } A \text{ of } T \text{ wrt } \mathcal{B} \text{ is diagonal.}

\begin{bmatrix}
\lambda_1 & 0 & 0 & \cdots & 0 \\
0 & \lambda_2 & 0 & \cdots & 0 \\
0 & 0 & \lambda_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \lambda_n
\end{bmatrix}

\text{Def:}
Suppose $T : \mathcal{V} \to \mathcal{V}$ is diagonalizable. Then there is a basis of $\mathcal{V}$ of $e$-vectors.

Conversely, also have

If there is basis of $e$-vectors for $T$, previous Theorem says...
$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

Example: $V = \mathbb{R}^2$, $T$ given by matrix $A$, $T$ is diagonalizable.

Claim: $N^c T$ all lie trans. $T: V \rightarrow V$.
Step (1) Find e-values.

Let's see why $L$ is not diagonalizable.

$v$-vectors.

Roots: $av^2 + bv + c = 0$ with $2$. 

$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. 

$A(\pm t) = (-t)^2 = t^2$.
So there is not a basis of e-vectors. 

Scales of \{f_1, f_2\} are non-zero e-vectors. All other e-vectors are not e-vectors.

\[
E^o = \text{Span} \{1, 1\} = N^o \text{null}(A) = N^\perp \text{null}(A) \\
E_2 = \text{null}(A - 0.5I) = N^o \text{null}(A)
\]

Steps: 1. \[\text{null}(A) \] or 2.

\[\text{null}(A - 0.5I) \] or 1 or 2.
Step (2) \[ \Delta \text{in } E, \Delta \text{in } E^2 = \Delta \text{in } E^2 = F. \]

Set \( n \) with \( n = 1, 2 \) values:

\[ A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \]

Diagonalizable? Given by matrix \( A \) with eigenvalues \( \lambda \in \mathbb{R} \).

More examples
Take basis $\beta = \{y_1, y_2\}$

$E_2 = \text{Null}(A - 2I) = \text{Null} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \text{span} \{y_1\}$

$E_1 = \text{Null}(A - 1.1I) = \text{Null} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \text{span} \{y_2\}$

Find $e$-vectors:
\[ \text{Step 1: } \mathbf{A} \mathbf{x}(t) = (2-t)(1-t) \mathbf{x} \]

Diagonalize by matrix \( \mathbf{V} \):

\[ \mathbf{V} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]

The given matrix has the following properties:

1. \( n = 2 \)
2. \( \mathbf{A} \in \mathbb{R}^2 \), \( \mathbf{x} \in \mathbb{R}^2 \) give by matrix.
Calc $E_1 = \text{null } (A - I) \begin{bmatrix} 1 & 0 & 0 \\ \end{bmatrix}$

$\alpha = \frac{1}{2}$  \hspace{1cm} $\gamma = \frac{1}{2}$

$\text{dim } E_1 = 1$ or 2

Not diagonalizable

Step 2
Wnt $Z \text{ dim } E_2 = \text{dim } V$
Else, not.

Always factor X(e) complex roots

Next time we'll set

Step 1: Find roots of X(a)

Step 2: Find e-spaces

Step 3: Find e-vectors

Step 4: Check if bases of e-vectors Equnuitarily

Then diagonalizable!

x-evalue

If a_{ii} \neq 2 \text{ for } i \neq j

Summary of Strategy to check if

T: v \rightarrow \text{ is diagonalizable.}