You have 20 minutes to complete this quiz. To receive full credit, you must justify your answers.

Name : ______________________

1. (5 points) What are the dimensions of the null space and column space of

$$A = \begin{bmatrix} 1 & -1 & 7 \\ 2 & 1 & 8 \\ 3 & 3 & 9 \end{bmatrix}$$

Solution: Augment A with the 0 vector and row reduce to find the dimension of the vector space of solutions to the equation $Ax = 0$. Row reducing, we obtain

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So, the dimension of the vector space of solutions to the equation $Ax = 0$ is 1. Hence, the dimension of the null space of A is 1. By the rank theorem, the dimension of the column space of A is equal to $2 = 3 - 1$.

2. (5 points) If an $m \times n$ matrix A has rank k, find the dimension of the null space of A^T.

Solution:

The column space of A^T is the row space of A, which has same dimension as the column space of A, which is k. By the rank theorem applied to A^T,

$$m = \dim \text{Nul } A^T + k$$

So,

$$\dim \text{Nul } A^T = m - k$$