Math 54
Fall 2017
Practice Exam 1
Exam date: 9/26/17
Time Limit: 80 Minutes

Name: ______________________

Student ID: ______________________

GSI or Section: ______________________

This exam contains 6 pages (including this cover page) and 7 problems. Problems are printed on both sides of the pages. Enter all requested information on the top of this page.

> This is a closed book exam. No notes or calculators are permitted.
> We will drop your lowest scoring question for you.

You are required to show your work on each problem on this exam. **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

If you need more space, there are blank pages at the end of the exam. Clearly indicate when you have used these extra pages for solving a problem. However, it will be greatly appreciated by the GSIs when problems are answered in the space provided, and your final answer **must** be written in that space.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

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1. (a) (5 points) Determine if $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 3 \\ -1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 4 \\ 8 \\ 16 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -3 \\ 2 \\ -3 \\ 2 \end{bmatrix}$.

(b) (5 points) Without doing any further computations, determine if $A = \begin{bmatrix} 3 & 2 & 0 & -3 \\ -1 & 4 & 0 & 2 \\ 2 & 8 & 1 & -3 \\ 3 & 16 & 0 & 2 \end{bmatrix}$ is invertible. Then, compute $\det(A)$ to verify your conclusion.

2. (a) (5 points) Suppose that the augmented matrix corresponding to a linear system of equations can be row reduced to the following augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 5 \\ 0 & 1 & 7 & -2 \end{array}\right]$$

Describe the solution set in parametric vector form. Then state what shape it represents geometrically; for example, your answer could be of the form: a plane, a sphere, etc.

(b) (5 points) Describe the span of $v_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$, and $v_3 = \begin{pmatrix} 3 \\ 4 \\ 10 \end{pmatrix}$ geometrically in the same way as in part (a).

How many vectors (if any at all) can be removed from the collection $\{v_1, v_2, v_3\}$ so that the resulting collection of vectors has the same span as $\{v_1, v_2, v_3\}$?

3. (10 points) Consider the matrices below.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 2 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 4 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix}$$

(a) (3 points) Compute $\det(AB)$.

(b) (3 points) Compute the determinant of the matrix C obtained from A by switching the first and third row, then adding twice the second column to the first, and then multiplying the second and third rows by 2.

(c) (4 points) Do A and B commute?

4. (10 points) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 - x_2 \\ -x_1 + x_2 \end{pmatrix}.$$

(a) (3 points) Compute the matrix of T.

(b) (2 points) Write the definition of what it means for T to be onto.

(c) (1 point) Is T onto?

(d) (2 points) Write the definition of what it means for T to be one-to-one.

(e) (2 points) Is T one-to-one?

5. (10 points) Label the following statements as True or False. The correct answer is worth 1 point. An additional point will be awarded for a correct brief justification. No points will be rewarded if it is not clear whether you intended to mark the statement as True or False.

   (a) (2 points) A homogeneous system is always consistent.

   (b) (2 points) Let RREF(C) be the reduced row echelon form of a matrix C. If A and B are matrices such that AB is well defined, then RREF(AB)= RREF(A)RREF(B).

   (c) (2 points) Assume A and B are invertible. If $AB = BA$, then $A^{-2}B^{-2} = B^{-2}A^{-2}$ where $A^{-2} = (A^{-1})^2$.

   (d) (2 points) If A is an invertible $n \times n$ matrix then $A^{-1} = \frac{1}{\det(A)}C$, where C is the matrix of A's cofactors.

   (e) (2 points) If $k < n$, then a set of k vectors in $\mathbb{R}^n$ is necessarily linearly independent.

6. (10 points) Using ■ to denote a leading entry, $*$ to denote an arbitrary nonleading entry, and 0 to denote an entry that must be 0, describe all 2×3 row echelon form (REF) matrices. (*Hint*: There are seven.)

   For each matrix, say whether it has linearly independent or linearly dependent columns and whether or not its columns span $\mathbb{R}^2$.

7. (10 points) Suppose that A is an invertible $n \times n$ matrix with integer entries.
   (a) (5 points) Show that if $\det(A) = \pm 1$, then A^{-1} has integer entries.

   (b) (5 points) Show that if A^{-1} has integer entries, then $\det(A) = \pm 1$. (*Hint*: Consider $\det(A^{-1})$.)

Extra space

Extra space.