Name: ______________________________

1. For each pair of homogenous ideals $I, J \subset k[x,y,z]$, consider the intersection of $X = \text{Proj}(k[x,y,z]/I)$ and $Y = \text{Proj}(k[x,y,z]/J)$ inside of $\mathbb{P}^2_k = \text{Proj}(k[x,y,z])$. Calculate the irreducible components of the intersection and the global functions on each irreducible component.

   1. $I = (x)$, $J = (x^3 - yz^2)$.

   2. $I = (y)$, $J = (x^3 - yz^2)$.

2. If k is algebraically closed and $X = \text{Proj}(k[x_0, x_1, \ldots, x_n]/I)$ is connected and reduced (no nilpotents in sections of its structure sheaf), then $\Gamma(X, \mathcal{O}_X) \simeq k$. But...

   Calculate $\Gamma(X, \mathcal{O}_X)$ for the following.

   1. $X = \text{Proj}(k[x_0, x_1]/(x_1^2))$.

   2. $X = \text{Proj}(\mathbb{R}[x_0, x_1]/(x_0^2 + x_1^2))$.

3. If $X = \text{Spec}(k[x_1, \ldots, x_n]/I)$ is dimension $n-1$ and $\Gamma(X, \mathcal{O}_X) = k[x_1, \ldots, x_n]/I$ is an integral domain, then X is cut out of $\mathbb{A}^n_k = \text{Spec}(k[x_1, \ldots, x_n])$ by a single equation. (In a unique factorization domain, every height one prime ideal is principal.) But...

   Give an example of an irreduble dimension 1 scheme $X = \text{Spec}(k[x_1, x_2]/I)$ that is not cut out of $\mathbb{A}^2_k = \text{Spec}(k[x_1, x_2]$ by a single equation. (So by necessity $\Gamma(X, \mathcal{O}_X)$ must have nilpotents.)