Name: ____

1. Prove that if X is a Noetherian space, and $U \subset X$ is a subspace, then U is a Noetherian space and hence quasi-compact.

2. For each $s \in \mathbb{C}$, consider the given ideal $I_s \subset \mathbb{C}[x, y, z]$ and quotient ring $R_s = \mathbb{C}[x, y, z]/I_s$. Decide whether $\mathrm{Spec} R_s$ is connected and whether it is irreducible.

    1. $I_s = (x^2 + y^2 - z^2, x - s)$.

    2. $I_s = (x^2 + y^2 - z^2, z - s)$.

3. Calculate the (Krull) dimension of $\mathrm{Spec} R$ for each given ring R.

    1. $R = \mathbb{C}[x]/(x^2)$.

    2. $R = \mathbb{C}[[x]]$.

    3. $R = \mathbb{C}[x, y, z]/(xy, xz)$.