1. (a) Let \( B = \{ b_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \} \) and \( C = \{ c_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, c_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix} \} \) be bases of \( \mathbb{R}^2 \).

Find the change-of-coordinates matrix from \( B \) to \( C \).

(b) Let \( x = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \). Using the fact that \( [x]_B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \), find the coordinates of \( x \) with respect to the \( C \) basis.

**Solution:** (a) P from \( B \) to \( C \) is \( \begin{bmatrix} 7/5 & 4/5 \\ -6/5 & -7/5 \end{bmatrix} \).

(b) Multiply \( P \) on the right by \( [x]_B \) to get \( [x]_C = \begin{bmatrix} 5 \\ -5 \end{bmatrix} \). A simple row reduction lets you use this answer to check your matrix from part (a). Make sure you see why this is true.

2. Find the eigenvalues of \( A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix} \) and one corresponding eigenvector for each eigenvalue.

**Solution:** The eigenvalues of \( A \) are 6 and \(-1\).

One corresponding eigenvector for \( \lambda = 6 \) is \( \begin{bmatrix} 4/3 \\ 1 \end{bmatrix} \). Scalar multiples of this vector except for the vector \( \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) would be acceptable eigenvectors.

A corresponding eigenvector in the eigenspace of \( A \) for \( \lambda = -1 \) is \( \begin{bmatrix} -1 \\ 1 \end{bmatrix} \).

Again, scalar multiples of this vector except for the zero vector are acceptable eigenvectors.