MATH 54 Quiz 1 28 August 2015

You have 20 minutes to complete the quiz. No calculators.

Name:_____

1. (5 points) Find all solutions of the following linear system:

$$\begin{aligned} -2x_1 + 2x_2 &= 4 \\ x_1 - x_2 &= -2 \end{aligned}$$

.

Solution. The first equation is -2 times the second equation. Thus, x_1, x_2 solves the system if and only if it solves the second equation. For any number t, the second equation is solved by $x_1 = t, x_2 = 2 + t$. □

2. (5 points) Consider the following linear system:

$$\begin{aligned} cx + y &= 5 \\ x + y &= 2 \end{aligned}$$

For what values of c does this system have no solutions? For which values of c does it have a unique solution? For which values of c does it have infinitely many solutions?

Solution. If $c = 1$, the system is

$$\begin{aligned} x + y &= 5 \\ x + y &= 2, \end{aligned}$$

which implies that $5 = 2$: a contradiction. Therefore, if $c = 1$, the system has no solution. So, suppose that $c \neq 1$.

Subtracting the second equation from the first yields,

$$\begin{aligned}(c-1)x &= 3\\ x+y &= 2.\end{aligned}$$

As $c \neq 1$, we may divide by $c-1$ to obtain $x = \frac{3}{c-1}$. The second equation then implies that $y = 2 - \frac{3}{c-1}$.

Thus, if $c = 1$ there is no solution and if $c \neq 1$ there is exactly one solution. In particular, for no value of c does the system have infinitely many solutions. □

The End.