1. Solve the given initial value problem:

\[ y''' - 5y'' + 8y' - 4y = 0 \]

given that

\[ y(0) = -1, \quad y'(0) = -3, \quad y''(0) = -6 \]

**Solution:**

Auxiliary Equation:

\[ r^3 - 5r^2 + 8r - 4 = 0 \]

\[ (r - 1)(r - 2)^2 = 0 \]

\[ r_1 = r_2 = 2, \quad r_3 = 1 \]

General Solution:

\[ y = c_1 e^t + c_2 e^{2t} + c_3 t e^{2t} \]

Take derivative and use initial conditions:

\[ \begin{cases} 
    c_1 + c_2 = -1 \\
    c_1 + 2c_2 + c_3 = -3 \\
    c_1 + 4c_2 + 4c_3 = -6 
\end{cases} \quad \Rightarrow \quad \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \]

Solution:

\[ y = 2e^t - 3e^{2t} + te^{2t} \]

2. Show that \( \{x, x^2, x^3, x^4\} \) are linearly independent on \( (-\infty, \infty) \).

**Solution:**

We can show directly without appealing to differential equations. Suppose \( c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 = 0 \). That is, it is identically zero for all \( x \). Then since a polynomial of degree \( n \), \( n > 0 \) has at most \( n \) roots, this must be a polynomial of degree 0, i.e. the constant polynomial 0. Thus \( c_1, c_2, c_3, c_4 = 0 \), so these functions are linearly independent.

Alternatively, notice that \( \{1, x, x^2, x^3, x^4\} \) are all solutions to \( y^{(5)} = 0 \), and apply the Wronskian method.

\[
W = \begin{vmatrix} 1 & x & x^2 & x^3 & x^4 \\ 0 & 1 & 2x & 3x^2 & 4x^3 \\ 0 & 0 & 2 & 6x & 12x^2 \\ 0 & 0 & 0 & 6 & 24x \\ 0 & 0 & 0 & 0 & 24 \end{vmatrix} = 1 \times 1 \times 2 \times 6 \times 24 = 288 \neq 0
\]

Since the Wronskian is nonzero, the functions \( \{1, x, x^2, x^3, x^4\} \) are linearly independent on \( (-\infty, \infty) \), and so is the subset \( \{x, x^2, x^3, x^4\} \) linearly independent on \( (-\infty, \infty) \).