Name: ______________________________

You have 20 minutes to complete the quiz.

1. Solve the given initial value problem:

$$y''' - 5y'' + 8y' - 4y = 0$$

given that

$$y(0) = -1, y'(0) = -3, y''(0) = -6$$

Solution: Auxillary Equation:

$$r^3 - 5r^2 + 8r - 4 = 0$$
$$(r-1)(r-2)^2 = 0$$
$$r_1 = r_2 = 2, r_3 = 1$$

$$\text{General Solution:} \quad y = c_1 e^t + c_2 e^{2t} + c_3 t e^{2t}$$

$$\text{Take derivitive and use initial conditions:} \quad \begin{cases} c_1 + c_2 = -1 \\ c_1 + 2c_2 + c_3 = -3 \\ c_1 + 4c_2 + 4c_3 = -6 \end{cases} \Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$\text{Solution :} \boxed{y = 2e^t - 3r^{2t} + te^{2t}}$$

2. Show that $\{x, x^2, x^3, x^4\}$ are linearly independent on $(-\infty, \infty)$.

Solution: We can show directly without appealing to differential equations. Suppose $c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 = 0$. That is, it is identically zero for all x. Then since a polynomial of degree n, $n > 0$ has at most n roots, this must be a polynomial of degree 0, i.e. the constant polynomial 0. Thus $c_1, c_2, c_3, c_4 = 0$, so these functions are linearly independent.

Alternatively, notice that $\{1, x, x^2, x^3, x^4\}$ are all solutions to $y^{(5)} = 0$, and apply the Wronskian method.

$$W = \begin{vmatrix} 1 & x & x^2 & x^3 & x^4 \\ 0 & 1 & 2x & 3x^2 & 4x^3 \\ 0 & 0 & 2 & 6x & 12x^2 \\ 0 & 0 & 0 & 6 & 24x \\ 0 & 0 & 0 & 0 & 24 \end{vmatrix} = 1 \times 1 \times 2 \times 6 \times 24 = 288 \neq 0$$

Since the Wronskian is nonzero, the functions $\{1, x, x^2, x^3, x^4\}$ are linearly independent on $(-\infty, \infty)$, and so is the subset $\{x, x^2, x^3, x^4\}$ linearly independent on $(-\infty, \infty)$.