Albert Einstein

into lazy habits of thinking. A man who reads too much and uses his own brain too little falls

"Any man who reads too much and

Friday: Quiz through Quiz

Orthogonal Linear
0 = h \left( \frac{\partial h}{\partial x} - \frac{\partial ^2 h}{\partial x^2} \right)

0 = \frac{\partial h}{\partial x} - \frac{\partial h}{\partial t} + \sin(t)

Solve IVP

\text{Solve homogeneous even}

\text{Exercises}

\begin{align*}
(0, 0, 0, 0) & = (0, 0, 0, 0) \\
(0, h, h, 1) & = (0, h, h, 1) \\
(0, -h, 1, 0) & = (0, -h, 1, 0)
\end{align*}
Basis for solutions $t = e^t, e^t, e^{-t}$

$$0 = y = (\frac{dy}{dt} I + I) (\frac{d^2y}{dt^2} I - I)$$

Homogeneous factors

Roots: $r = 0, 1, -1$

Factor $r (r - 1) (r + 1) = 0$

Auxiliary equation $r_1^2 - r = 0$
\[ t \begin{array}{c} \text{span} \\ \text{t is in} \end{array} \left\{ t \begin{array}{c} 1 \\ \text{guess} \end{array} t - 2t \right\} \]

\[ y = y, \quad y' = \sin(t) \]

Guess \[ A \cos(t) + B \sin(t) \]

Solve \[ y', \quad y = \sin(t) \]

Take: \[ y = -\frac{2}{1+t^2} \]

Back to nonhomog even
to homogenous

Gen. soln

\[ y = - \frac{2}{3} t \left( e^t + \frac{1}{2} \cos t \right) + C \]

Gen. soln to nonhomogenous

\[ y = \frac{1}{2} \cos t \]

Take \( y = \frac{1}{2} \cos t \) \( A = \frac{1}{2}, B = 0 \)
To solve $Tx = \vec{b}$, find a solution and then add any general solution.

Reminder: To solve $Tx = \vec{b}$, find $Tx = \vec{0}$.
This is a nonhom. lin syst.

For $c_1, c_2, c_3$:

$0 = y(0) = \frac{2}{3} + c_2 + c_3$

$0 = y'(0) = c_2 - c_3$

$2 = \overline{\text{want}}$

$y''(0) = \frac{2}{1} + c_1 + c_2 + c_3$

Now IVP: Evaluate $y(0), y'(0), y''(0)$
\[ 0, 1, c_1 = \frac{h}{3}, c_2 = \frac{h}{3}, c_3 = \frac{h}{3} \]

\[
\begin{pmatrix}
\frac{2}{3} - 1 & 0 \\
0 & -1 & \frac{2}{3} & c_3 \\
\frac{2}{3} & \frac{1}{3} & 1 & c_2
\end{pmatrix}
= 
\begin{pmatrix}
1 & 1 & 0 \\
1 & -1 & 1 & 0 \\
1 & 1 & 1 & 1
\end{pmatrix}
\]

Organize as matrix eqn

and solve
reduce
row
←
Where do I start?

\[ y = -\frac{2}{3} t^2 + \frac{4}{3} t e^t + \frac{1}{3} e^{-t} \]

Solve to IVP:
Theorem: True on entire line \( R \)

and \( x_0, y_0, \ldots, y_{n-1} \) are numbers

where \( \beta_1, \ldots, \beta_n \) are functions

with \( y(x_0) = y_0, y(x_1) = y_1, \ldots, y(x_n) = y_n \)

so \( \beta = \beta_1 + \beta_2 + \cdots + \beta_n \) is the unique general IVP solution.
Basis of solutions: \( y_1, \ldots, y_n \)

Homogeneous: \( Ty = 0 \)

\[
I = \sum_{i=1}^{n} \left( \frac{a_i}{\lambda_i} \right) y_i + \cdots + p_{n-1}\lambda^{n-1}
\]

If transp. \( T : V \leftarrow V \)

Useful analysis: \( V \) vec sp of \( f \)
Now need to solve IVP

Gen soln: \[ y = y_0 + c_1 y_1 + \ldots + c_n y_n \]

Nonhomog eqn: 

There always is a single soln \( y \). 

\[ Ty = g \]
A unique soln?

Why is there always

\[
\begin{bmatrix}
 x_0 \\
 x_1 \\
 x_2 \\
 \vdots \\
 x_n \\
\end{bmatrix}
 =
 \begin{bmatrix}
 c_0 \\
 c_1 \\
 c_2 \\
 \vdots \\
 c_n \\
\end{bmatrix}
 \begin{bmatrix}
 y_0(x_0) \\
 y_1(x_1) \\
 y_2(x_2) \\
 \vdots \\
 y_n(x_n) \\
\end{bmatrix}
\]

Solve lin syst
$\text{W} \left[ y_1, \ldots, y_n \right](t) \neq 0$ \iff invertible at $t$. 

$\begin{bmatrix}
  y_1'(t) \\
  y_2'(t) \\
  \vdots \\
  y_n'(t)
\end{bmatrix}
= \frac{d}{dt} \text{W} \left[ y_1, \ldots, y_n \right](t)
\text{Wronskian in Weyl group}$

$\text{Definition for } y_1, \ldots, y_n$.
Ex: Calc Wronskian for fns from warmup problem $y_1 = 1, y_2 = e^t, y_3 = e^{-t}$

$\text{Soln } W(y_1, y_2, y_3)(t) = \det \begin{bmatrix} 1 & e^t & e^{-t} \\ 0 & e^t & -e^{-t} \\ 0 & e^t & e^{-t} \end{bmatrix} = 2$

in particular $\neq 0$ for any $t$
1) \( y(0) \neq 0 \) (solve IVP at \( x_0 \))

2) \( y(1) \neq 0 \) (solve IVP at any \( t \))

3) \( y(1) \ldots y(n) \neq 0 \) (any \( t \))

\[
\frac{d^ny}{dt^n} + \cdots + p_{n-1} y^{(n-1)} + p_n y = 0
\]

Weinstein Lemma. The following are equivalent:

For any \( y_1, \ldots, y_n \) that solve
\[ M(y', y'') + \lambda = 0 \] 

Then the version lemma says

If they solve \( y' + p'y' + p_2 = 0 \), then \( y_1, y_2 \) are linearly independent.

\[ \overline{\text{Seln } y_1, y_2 \text{ are lin indep.}} \]

To solve \( y' + p'y' + p_2 = 0 \), can \( y_1 = 1, y_2 = e^t \) be solutions?

\[ \overline{\text{Exer}} \]
If given $y^2 = -1$, cannot both solve 2nd order

$$0 = 0 \quad \text{when} \quad t = 0$$

$$= 2te^{2t}$$

$$= \left[ \begin{array}{cc} 2te^{2t} & 0 \\ 0 & 1 \end{array} \right] = \cosh \left( t \right)$$
Now specialize to constant coeff. gen's
Solve

\[ e^{\sin(t)} \]

or alternatively:

\[ e^{\cos(st)} \]

1) For real roots: if \( e^{\pm i\phi} \) is a root of an even form of solutions, depends on roots of solutions can be found explicitly.

2) \( r = x + iy \) (complex root: \( e^{(x+iy)t} \)

\( r = i \pm 1 \)}
can combine cases \(1 \& 3\)

Think: If complex root repeats, then

solve

3) repeated root \(a\) of multiplicity \(k\) it, et al., the...
Basis of solns \(1, t, te^t\)

Roots 0, 1, -1

Factors 1 - 1 = 1 - t + 1 = (t - 1)(t + 1)

Solve aux eqn \(r - 4 = 0\)

Solve \(\frac{dy}{dx} = y(1 - y(2))\) = 0