Next week:
Office Hours: Tuesday
1-3 pm, Location TBA

This week:
Friday
"Practice Quiz"

Enrollment issues: Thomas Brown

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An eigenvalue is at heart a linear algebra algorithm. It's easy! For example, Google's PageRank uses eigenvalues! It's powerful! Solve eigenvalues by linear algebra. It's fun! It's easy! It's powerful! Why take math shy?
Notation

\[ \mathbb{R} = \text{real numbers} \]

\[ \mathbb{C} = \text{complex numbers} \]

Polar form: \[ z = r \cos \theta + ir \sin \theta = re^{i \theta} \]

More notation: \( a, b, c, \ldots \) numbers, \( x, y, z, \ldots \) variables
\[ a_1 x_1 + a_2 x_2 + \ldots + a_n x_n = b \]

The equation can be put in the form:

\[ A \vec{x} = \vec{b} \]

A linear equation in n variables is an even
Key idea of linear sums and again solutions.

if the number $b = 0$.

Scales of solutions are again solutions.

1) $3(x_1 - x_3) = 5(x_4 - x_3)$
2) $3(x_1^2 - 2x_2^2) = \pm 7$
3) $(x_1 - 3)^2 = x_2^2 + 5x_2$

Which of the following is a linear eqn?

Yes

No
Def A system of \textit{lin eqns} / \textit{linear syst.} in \( n \) vars is a finite collection of \( \text{lin eqns} \).

Can be put in form:

\[
\begin{align*}
& a_{11} x_1 + \ldots + a_{1n} x_n = b_1 \\
& \vdots \\
& a_{m1} x_1 + \ldots + a_{mn} x_n = b_m
\end{align*}
\]
Examples of linear systems:

1) \( m=2, n=3 \)

2) \( m=3, n=1 \)

\[ x_1 - x_2 = 2 \]
\[ 3x_1 = 0 \]
\[ 2x_1 - 3x_2 + 7x_3 = 5 \]
\[ -x_1 + 0x_2 - 4x_3 = 0 \]

\( m < n \Rightarrow m > n \Rightarrow \text{easier to solve} \)

Note: \( n > m \)

Harder to solve
Def: Solution set of a lin. syst. is the set of all \((s_1, \ldots, s_n)\) that solve all the eqns. "n-tuple" of numbers

Exer: Find soln set for

1) \(3x_1 - x_2 = 0\)

   \(2x_1 = 6\)

Soln set = \(\{(3, 9)\}\)

There is unique soln.
2. \[3x_1 + x_2 = 1\]

3. \[x_1 - x_2 + x_3 = 0\]

\[x + x_2 + x_3 = 0\]

Solutions to set:

\[-6x_1 - 2x_2 = 0\]

\[x = 2, x_1, x_2 = 0\]

Solution set = \[\emptyset\]

No solution.

Let \(s\) be any number,s

\[f(-2s, -s, s)\]
$2x^1 + 2x^2 = 0$

the above: $x^1 + cx^2 = 1$

Exer. For what number $c$ does the following lin syst satisfy each of the following?

1) No solns. - inconsistent
2) Unique soln.
3) Infinitely many solns. - consistent

Three possible outcomes:
Solve for \( s \):

\[
\frac{2-1}{1} \cdot (2-1) = 1
\]

Then let \( s + c(s-5) = 1 \) and begin solving for \( s \). Any solution to \( s \) and \( c \) will imply a solution.
If $c = 1$, then there is no solution.

If $c \neq 1$, then there is a unique solution $\left\{ \frac{c}{1-c}, \frac{1}{1-c} \right\}$.

Conclusion: $c \neq 1$, then there is
The number of solutions is given by the set \( \{ (c, s) : \text{for } I \text{ and } \Sigma \text{, we have } x_1 + x_3 = 0 \} \) \( \cap \) \( \{ (c, s) : c \in (-\ell, -\ell + \ell) \} \).

\[
\begin{align*}
x_2 + x_3 &= 0 \\
x_1 - c x_2 &= 0 \\
2 x_1 - x_2 + x_3 &= 0
\end{align*}
\]

In systems equivalent? For what \( c \) are the following equivalent?

For two linear systems, \( \text{they have the same solution sets.} \)
Take $t = -5$. 

Some number $\{ (t, t) \}$ = Some number $\{ (s, s - 5) \}$

$1 = c$.

Conclusion: sets are same.
Matrix Notation:

\[
\begin{bmatrix}
2x_1 - 3x_2 + 0x_3 - x_4 = 7 \\
x_1 + x_2 - 3x_3 + 2x_4 = 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
2 & -3 & 0 & -1 \\
1 & 1 & -3 & 2
\end{bmatrix}
\]

\[m = \begin{bmatrix} 2 & -3 & 0 & -1 \end{bmatrix}
\]

\[n+1=5\text{ cols}
\]

Called: augmented matrix of lin syst.