Name (Last, First): ______________________________

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1) Consider the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Find a basis for the eigenspace with eigenvalue $\lambda = 1$.

Solution: The eigenspace of A corresponding to an eigenvalue λ is the null space of the matrix $A - \lambda I$. For $\lambda = 1$, we have

$$A - \lambda I = A - I = \begin{bmatrix} 3 & 1 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

A vector $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a solution to $(A - I)\mathbf{x} = \mathbf{0}$ if and only if

$$3x_1 + x_2 + x_3 = 0 \qquad -2x_1 = 0$$

so if and only if

$$x_1 = 0 \qquad x_3 = -x_2$$

Thus the general solution is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \\ -x_2 \end{bmatrix} = x_2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

Thus the eigenspace is one-dimensional with basis

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

2) Find the characteristic equation and the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$$

Solution:

$$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 - 4 = (1-\lambda-2)(1-\lambda+2) = (-1-\lambda)(3-\lambda)$$

Thus the characteristic equation is

$$(-1-\lambda)(3-\lambda) = 0$$

and the eigenvalues are -1 and 3.