Name (Last, First): ______________________________

Student ID: ______________________________

Circle your GSI and section:

Scerbo	8am	200 Wheeler
Scerbo	9am	3109 Etcheverry
McIvor	12pm	3107 Etcheverry
McIvor	11am	3102 Etcheverry
Mannisto	12pm	3 Evans
Wayman	1pm	179 Stanley
Wayman	2pm	81 Evans
Forman	2pm	3109 Etcheverry
Forman	4pm	3105 Etcheverry
Melvin	5pm	24 Wheeler
Melvin	4pm	151 Barrows
Mannisto	11am	3113 Etcheverry
McIvor	2pm	179 Stanley

If none of the above, please explain: ______________________________

This is a closed book exam, no notes allowed. It consists of 6 problems, each worth 10 points, of which you must complete 5. **Choose one problem not to be graded by crossing it out in the box below.** You must justify every one of your answers unless otherwise directed.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total Possible	50	

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1. a. State what it means for a list of vectors $v_1, \ldots, v_n$ in a vector space V over a field F to be linearly independent.

b. Consider the real vector space $P_{\leq 2}(\mathbb{R})$ of polynomials of degree less than or equal to two with real coefficients.

For what vectors $p(x) \in P_{\leq 2}(\mathbb{R})$ is the list $p(x), p'(x), p''(x)$ linearly independent? (Here $p'(x)$ denotes the derivative of $p(x)$, and $p''(x)$ denotes the second derivative of $p(x)$.)

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2. Suppose $T : V \to W$ is an injective linear transformation of finite-dimensional vector spaces. Show there exists a linear transformation $S : W \to V$ such that the composition $ST : V \to V$ is the identity transformation.

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3. Let $P(\mathbb{C})$ be the complex vector space of polynomials with complex coefficients.

Consider the map $T : P(\mathbb{C}) \to \mathbb{C}$ that takes a polynomial $p(z) = a_0 + a_1 z + \cdots + a_n z^n$ to the sum of the complex conjugates of its coefficients

$$T(p(z)) = \bar{a}_0 + \bar{a}_1 + \cdots + \bar{a}_n$$

Is T linear? Be sure to justify your answer.

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4. Let V be a two-dimensional vector space. Suppose $T : V \to V$ is a linear transformation that is not a scalar multiple of the identity. Prove that there exists a vector $v \in V$ such that the pair of vectors v, Tv form a basis of V.

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5. Decide if the following assertion is always true or sometimes false. If always true, provide a proof; if sometimes false, provide a counterexample and justify why it is a counterexample.

Assertion: Let V be a finite dimensional complex vector space, $T : V \to V$ a linear transformation, and $U \subset V$ a T-invariant subspace. Then there exists a T-invariant subspace $W \subset V$ such that $V = U \oplus W$.

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6. Let V be a vector space of dimension 3, and let W be a vector space of dimension 5. Recall that $L(V, W)$ denotes the vector space of linear transformations from V to W.

Show that there cannot exist a linear transformation

$$T : L(V, W) \to L(V, W)$$

such that $\dim \operatorname{null}(T) = \dim \operatorname{range}(T)$.