Name (Last, First): ________________________________

Student ID: ________________________________

Circle your GSI and section:

Sparks	8am	105 Latimer
McIvor	9am	55 Evans
Hening	10am	7 Evans
Hening	11am	3113 Etcheverry
Sparks	12pm	285 Cory
Sparks	1pm	285 Cory
McIvor	2pm	3107 Etcheverry
McIvor	3pm	3107 Etcheverry
Tener	4pm	79 Dwinelle
Tener	5pm	81 Evans

If none of the above, please explain: ________________________________

This is a closed book exam, no notes allowed. It consists of 6 problems, each worth 10 points, of which you must complete 5. **Choose one problem not to be graded by crossing it out in the box below.** You must justify every one of your answers unless otherwise directed.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total Possible	50	

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1. Find linear transformations $S, T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $ST = 0$ but $TS \neq 0$. Prove that the rank of S and T must be 1.

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2. Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given in the standard basis by the matrix

$$\begin{pmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \in M_{2\times 3}(\mathbb{R})$$

Find a basis $\beta = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ such that with respect to the basis $\beta = \{v_1, v_2, v_3\}$ of $\mathbb{R}^3$ and the standard basis $\gamma = \{(1,0),(0,1)\}$ of $\mathbb{R}^2$, the matrix of T takes the form

$$[T]_\beta^\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \in M_{2\times 3}(\mathbb{R})$$

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3. Let V be a vector space with a finite basis $\beta = \{v_1, \ldots, v_n\}$.
   Define the dual space V^* and the dual basis $\beta^* = \{v_1^*, \ldots, v_n^*\}$.
   Calculate the matrix $[T]_\beta^\beta$ of the linear transformation $T : V \to V$ defined by

$$T(v) = v_1^*(v)v_1 + \cdots + v_n^*(v)v_n, \text{ for any } v \in V$$

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4. Decide if each of the following statements is always TRUE or sometimes FALSE. If always true, provide a proof. If sometimes false, provide a counterexample.

a) Suppose $A \in M_{m\times n}(F)$ with $m \geq n$. If $Ax = 0$ has exactly one solution, then $Ax = b$ has exactly one solution for any $b \in F^m$.

b) Suppose $A \in M_{m\times n}(F)$ with $m \leq n$. If $Ax = 0$ has exactly one solution, then $Ax = b$ has exactly one solution for any $b \in F^m$.

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5. Consider the vector space $P_2(\mathbb{R})$ of real polynomials of degree ≤ 2. For each of the following functions $f : P_2(\mathbb{R}) \to \mathbb{R}$, decide whether f is linear or not, justify your answer, and when it is linear, find a basis for its null space $N(f)$.

a) $f(p(x)) = p(1)$.

b) $f(p(x)) = p''(0)$.

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6. State what it means for two matrices $A, B \in M_{n\times n}(F)$ to be similar. Prove that if A and B are similar, then $A^2 - A + I_n$ and $B^2 - B + I_n$ are also similar, where $I_n \in M_{n\times n}(F)$ is the identity matrix.