Name (Last, First): ____________________
Student ID: ____________________

Circle your GSI and section:

Sparks	8am	105 Latimer
McIvor	9am	55 Evans
Hening	10am	7 Evans
Hening	11am	3113 Etcheverry
Sparks	12pm	285 Cory
Sparks	1pm	285 Cory
McIvor	2pm	3107 Etcheverry
McIvor	3pm	3107 Etcheverry
Tener	4pm	79 Dwinelle
Tener	5pm	81 Evans

If none of the above, please explain: ____________________

This exam consists of 10 problems, each worth 10 points, of which you must complete 8. **Choose two problems not to be graded by crossing them out in the box below.** You must justify every one of your answers unless otherwise directed.

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total Possible	80	

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1. Let $A \in M_{4\times 4}(\mathbb{C})$ with eigenvalues $-1, 2, 3$ with respective multiplicities $m_{-1} = 1, m_2 = 2, m_3 = 1$. Calculate the determinant of A. Be sure to justify your answer.

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2. Consider the matrix

$$A = \begin{pmatrix} 3/2 & 2 \\ -1 & -3/2 \end{pmatrix} \in M_{2\times 2}(\mathbb{R})$$

(a) Find the eigenvalues and corresponding eigenvectors of A.

(b) Show that for $v = (1,0) \in \mathbb{R}^2$, we can choose n large enough so that $\|A^n v\|$ is as small as we like.

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3. Suppose V is a finite-dimensional complex vector space, and $T : V \to V$ is a linear transformation such that $T^2 = T$.

(a) Prove that each eigenvalue of T is either 0 or 1.

(b) Prove that T is diagonalizable.

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4. Let V be a finite-dimensional real vector space with inner product $\langle \cdot, \cdot \rangle$.

(a) Fix $v \in V$, and consider the map

$$\varphi_v : V \to \mathbb{R} \qquad \varphi_v(w) = \langle v, w \rangle$$

Show that φ_v is linear, so in other words, φ_v is an element of the dual space V^*.

(b) Now consider the map

$$\Phi : V \to V^* \qquad \Phi(v) = \varphi_v$$

Show that Φ is linear and in fact an isomorphism.

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5. Let V be an inner product space and $S \subset V$ a subset.

(a) State what it means for S to be linearly independent. State what it means for S to be orthonormal.

(b) Prove that if S is orthonormal, then S is linearly independent.

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6. Let V be a finite-dimensional complex inner product space, and $S : V \to V$ a linear transformation. Let $S^* : V \to V$ be the adjoint of S, and $T : V \to V$ the composition $T = S^*S$.

(a) Prove that T is diagonalizable.

(b) Prove that all of the eigenvalues of T are non-negative real numbers.

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7. Let $A \in M_{n\times n}(\mathbb{C})$ be a complex matrix. Consider the subspace $W \subset M_{n\times n}(\mathbb{C})$ given by

$$W = span\{I, A, A^2, A^3, \ldots, A^k, \ldots\}$$

Show that

$$\dim W \leq n.$$

(Note that $\dim M_{n\times n}(\mathbb{C}) = n^2$ so we have $\dim W \leq n^2$, but you are being asked to show the stronger inequality $\dim W \leq n$.)

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8. Let $V = P_1(\mathbb{R})$ be the real vector space of degree ≤ 1 polynomials with inner product

$$\langle f, g \rangle_V = \int_0^1 f(x)g(x)dx$$

(a) Apply Gram-Schmidt to the basis $\{1, x\}$ to obtain an orthonormal basis.

(b) Find an isomorphism

$$T : V \to \mathbb{R}^2$$

such that for all $f, g \in V$, we have

$$\langle f, g \rangle_V = \langle Tf, Tg \rangle_{Euc}.$$

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9. Suppose $A \in M_{7\times 7}(\mathbb{C})$ is a matrix with characteristic polynomial

$$\chi_A(t) = (2-t)^2(3-t)^2(4-t)^3$$

Suppose as well that

$$\dim N(A-2I) = 1 \qquad \dim N(A-3I) = 2 \qquad \dim N(A-4I) = 1$$

Find a matrix which is a Jordan canonical form for A. Be sure to justify your answer.

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10. Find a basis for $\mathbb{C}^3$ that puts the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

into Jordan canonical form. What is the Jordan canonical form?