Math 110 Fall 2019 - Quiz 8 - BANK

date - time **Name:**

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(1) (5 points) Let γ be a cycle of generalized eigenvectors. Prove that the vectors in γ are linearly independent.

(2) (5 points) Let γ be a cycle of generalized eigenvectors for a linear transformation T. Show that the $\text{Span}(\gamma)$ is invariant under T.

(3) (5 points) Let A be a matrix over $\mathbb{C}$ whose only eigenvalue is $\lambda = 0$. Prove that A is nilpotent.

(4) (5 points) Compute the characteristic polynomial of A and determine how many eigenvectors it has given the following Jordan form,

$$[A]_\beta = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(5) (5 points) Compute the characteristic polynomial of A and determine how many eigenvectors it has given the following Jordan form,

$$[A]_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

(6) (5 points) Compute the characteristic polynomial of A and determine how many eigenvectors it has given the following Jordan form,

$$[A]_\beta = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$