

- (1) Determine whether the following matrix is diagonalizable over \mathbb{C} .

$$M = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 2 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

- (2) Determine whether the following matrix is diagonalizable over \mathbb{C} .

$$M = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- (3) Determine whether the following matrix is diagonalizable over \mathbb{C} .

$$M = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

- (4) Let A and B be matrices with the same characteristic polynomial over the same field. If A is diagonalizable, it is necessarily true that B is diagonalizable? Prove or give a counter example.

- (5) Let $T : V \rightarrow V$ be a diagonalizable linear transformation. Is T an isomorphism? Prove or give a counter example.

- (6) Let A and B be diagonalizable $n \times n$ matrices with the same set of eigenvectors. Show that A and B commute (that is, $AB = BA$).