Math 110 Fall 2019 - Quiz 5 - BANK

date - time **Name:**

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(1) Give linear transformation $S:\ V_1 \to V_2$ and $T:\ V_2 \to V_3$ such that $S \circ T$ is an isomorphism, but separately S and T are not. (You may choose V_1, V_2, and V_3 to be any vector spaces you like.)

(2) Define a linear transformation $T:\ V \to V$ such that T^2 is the zero transformation but T is not.

(3) Describe a linear transformation $T:\ \mathbb{R}^2 \to \mathbb{R}^2$ such that T is it's own inverse but T is not the identity transformation or a multiple of it.

(4) Let $T:\ V \to W$ and $S:\ W \to V$ be linear transformations such that $S \circ T$ is an isomorphism. Prove $\dim V \leq \dim W$.

(5) Let $T:\ V \to W$ and $S:\ W \to V$ be linear transformations. Prove the following,

$$\operatorname{rank}(S \circ T) \leq \min\{\operatorname{rank}(S), \operatorname{rank}(T)\}$$

(6) Let $S, T:\ V \to W$ be linear transformations. Prove the following,

$$\operatorname{rank}(T + S) \leq \operatorname{rank}(T) + \operatorname{rank}(S)$$