Math 110 Fall 2019 - Quiz 4 – BANK

date - time **Name:**

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(1) Show that $T:\ P_3(\mathbb{R}) \to P_3(\mathbb{R})$ is a linear transformation and find a basis for both $N(T)$ and $R(T)$.

$$T\left(p(x)\right) = xp'(x) + p(1)$$

(2) Show that $T:\ \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation and find a basis for both $N(T)$ and $R(T)$.

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 + 2x_2 \\ x_1 + 3x_2 \end{pmatrix}$$

(3) Show that $T:\ \mathbb{R}^2 \to P_2(\mathbb{R})$ is a linear transformation and find a basis for both $N(T)$ and $R(T)$.

$$T\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1x^2 + a_2x + (a_1 + a_2)$$

(4) Let $T:\ V \to V$ be a linear transformation of a finite vector space such that $\dim V = n$ and let W_1 and W_2 be subspaces of V such that $T(W_1) = T(W_2)$ and $m = \dim W_1 \geq \dim W_2$. Give examples of the following: (1) $\operatorname{rank}(T) = n$ and (2) $\operatorname{rank}(T) = n - m$.

(5) Let $V = W \oplus Z$ be a finite dimensional vector space (of dim n) and let W be a subspace of V. The projection of V onto W can be represented by an $n \times n$ matrix P. The matrix $I - P$ also represents a projection. Determining what subspace $I - P$ projects onto.

(6) Let V and W be finite dimensional vector spaces. Prove that if $T:\ V \to W$ is one-to-one then $\dim(V) \leq \dim(W)$. Give an example of why the converse is not true (that is show if $\dim(V) \leq \dim(W)$ that it is not necessarily true that T is one-to-one).