

- (1) Give a basis for the subspace of  $V = \mathbb{R}^3$  that satisfies the equations,

$$2x - y = 0$$

$$x + y - z = 0$$

- (2) What conditions must be placed on  $a, b, c, d, e$  and  $f$  such that the subset,

$$S = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right\} \subset \mathbb{R}^3$$

is linearly independent?

- (3) Prove that two perpendicular vectors in  $\mathbb{R}^2$  form a linearly independent set.

- (4) Let  $A$  be a  $3 \times 3$  skew symmetric matrix, that is  $A^T = -A$ . Prove or disprove: the columns of  $A$  form a linearly independent set.

- (5) Let  $S = \{s_1, \dots, s_n\}$  and  $T = \{t_1, \dots, t_m\}$  be linearly independent subsets of a vector space  $V$ . Prove or disprove: The set  $\{s_1, \dots, s_n, t_1, \dots, t_m\}$  is linearly independent if and only if  $\text{Span}(S) \cap \text{Span}(T) = \{0\}$ .

- (6) Describe a vector space,  $V$ , with a basis that contains infinitely many elements. Give an example of a basis of  $V$ .