Math 110 Fall 2019 - Quiz 2 BANK

date - time **Name:**

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(1) Show that

$$W = \left\{ \begin{pmatrix} x \\ 3x \\ 0 \end{pmatrix} \mid x \in \mathbb{R} \right\}$$

is a subspace of $V = \mathbb{R}^3$ with the usual componentwise addition and scalar multiplication.

(2) Give a counter example to the following statement: Let W_1 and W_2 be subspaces of a vector space V, then $W_1 \cup W_2 = \{x \mid x \in W_1 \text{ or } x \in W_2\}$ is also a subspace of V.

(3) Let $V = \mathcal{F}(\mathbb{R}, \mathbb{R})$ be the vector space of functions from the real numbers to the real numbers with addition and scalar multiplication defined by,

$$(f+g)(x) = f(x) + g(x) \quad \text{and} \quad (cf)(x) = c \cdot f(x)$$

A function f is 1-periodic if $f(x) = f(x+1)$ for all $x \in \mathbb{R}$. Show that the set of all 1-periodic functions is a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$.

(4) Consider the vector space,

$$V = M_3(\mathbb{R}) = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \mid a_{ij} \in \mathbb{R} \;\; \forall i, j \right\}$$

And consider the subspace of diagonal matrices,

$$D = \left\{ \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix} \mid a_{ii} \in \mathbb{R} \;\; \forall i \right\}$$

Find a subspace W of V such that: $W \neq V$, $W \neq D$, and $D \subset W$.

(5) Show that

$$W = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}$$

is a subspace of

$$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c \in \mathbb{R} \right\}$$

with the usual component-wise addition and scalar multiplication.

(6) Show that the subset of invertible matrices is not a subspace of $M_n(\mathbb{R})$ ($n \times n$ matrices with real components and component-wise addition and multiplication).