Math 110 Fall 2019 - Quiz 1 BANK

date - time **Name:**

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(1) Prove that the vector space $V = \mathcal{F}(S, \mathbb{R})$ for $S = \{0, 1\}$, with $(f+g)(x) = f(x) + g(x)$ and $c\dot{f}(x) = cf(x)$ has a zero element.

(2) A real valued function is odd if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. Show that the vector space $V = \{f(x) \in \mathcal{F}(\mathbb{R}, \mathbb{R}) \mid f(-x) = -f(x)\}$ is closed under the usual addition and scalar multiplication, that is

$$(f+g)(x) = f(x) + g(x), \quad (cf)(x) = c \cdot f(x)$$

(3) Describe a vector space (that is define V, F, addition and scalar multiplication) such that V has finitely many elements.

(4) Show that $V = \mathbb{R}$, $F = \mathbb{C}$, and the usual addition and scalar multiplication is not a vector space.

(5) Let V be the set of all infinite sequences with entries in $\mathbb{R}$,

$$V = \{x = (a_1, a_2, \dots, a_n, \dots) \mid a_i \in \mathbb{R} \forall i\}$$

Let $F = \mathbb{R}$ and define addition and scalar multiplication by the following:

$$x + y = (a_1, a_2, \dots) + (b_1, b_2, \dots) = (a_1 + b_1, a_2 + b_2, \dots)$$

$$c \cdot x = c \cdot (a_1, a_2, \dots) = (a_c, a_{c+1}, a_{c+2}, \dots, a_{c+n}, \dots)$$

Show that the above is not a vector space.

(6) Let $V = \{0, 1\}$, F be a field and define addition and scalar multiplication by,

$$0 + 1 = 1 + 0 = 1, \ 0 + 0 = 0, \ 1 + 1 = 0$$

$$c \cdot x = x \quad \forall x \in V \text{ and } c \in F$$

Prove this is not a vector space.