Final review
Math 32 Spring 2022 Name: _____

For questions 1 - 8 solve for all possible values of x. If the solution for x contains any interval, use proper interval notation. For extra practice, refer to section 0.2, 0.3, and 2.4 in the textbook.

1. $x - 4 = 3(x + 2)$

2. $\sqrt{x - 3} = 4$

3. $(x - 1)(x + 3) = -4$

4. $x^2 + 1 \geq 5$

5. $|2x - 4| = 3$

6. $|x - 1| > x$

7. $\left|\dfrac{x-1}{x+3}\right| \leq 3$

8. $|(x - 3)(x + 1)| = 2$

For questions 9 - 12 determine the domain and range of the following functions. When in doubt, sketch a graph of the functions. For extra practice, see sections 1.1, 1.4, and 2.4.

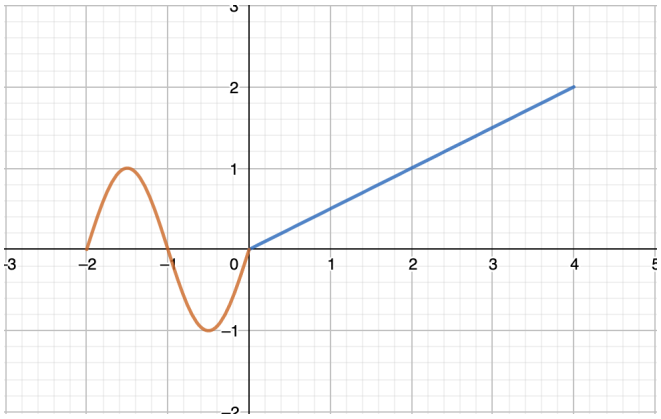
9. $f(x) = \sqrt{x-2}$

10. $f(x) = x^3 + x$

11. $g(t) = |t| - 3$

12. $g(x) = \dfrac{1}{x-4}$

Let $f(x)$ be the function depicted in the graph below. Assume the domain of $f(x)$ in the closed interval $[-2, 4]$. Answer the following questions regarding $f(x)$. For extra practice see sections 1.1 and 1.3.



14. What is the range of f? Express you answer in interval notation.

15. Let $g(x) = 2f(x)$. What is domain and range of $g(x)$?

16. Let $h(x) = -f(x-2)$. What is the domain and range of $h(x)$?

Questions 17 - 20 are about lines. For extra practice, see section 2.1 (and section 2.2 for 18).

17. Compute the slope-intercept form for the line that passes through the points $(1, 3)$ and $(4, -6)$.

18. Compute a line perpendicular to the line $y = \frac{1}{4}x + 3$ that passes through the vertex of the parabola $y = x^2 + x + 4$.

19. All points on a line have the form $(2t, 5t)$ for any real number t. What is the slope-intercept form of the line?

20. Consider the line through the points $(-3, 10)$ and $(2, 6)$. What is another point on the line?

Questions 21 - 24 are about polynomials. For extra practice, see sections 2.2 and 2.4.

21. Find the vertex of the parabola, $y = 2x^2 + 6x - 4$.

22. Find the where the parabola $y = x^2 + 5x - 3$ intersects the line $y = 2x + 10$.

23. Give an example of a polynomial (of any degree) whose range is exactly $[-1, \infty)$.

24. Give an example of a polynomial, $p(x)$, that satisfies the following: $\deg p(x) = 4$, $p(x)$ has a root at $x = 1$, and $p(2) = 2$.

The following question relate to sections 2.3, 3.1-3.3 and 3.5. See those sections for extra practice. For questions 25 - 30 solve for all possible x.

25. $\dfrac{3}{27^x} = 9^4$

26. $\log_x\left(\dfrac{1}{8}\right) = -3$

27. $\log_3(x-3) = 81$

28. $3^{x\log_3(7)} = 49$

29. $\log_2(8^x) = \log_2(24) - \log_2(6)$

30. $\ln(x+1) + \ln(x+3) = 0$

For the following questions, see sections 3.4 and 3.7 for practice.

31. A bacteria colony starts off with 20 cells. After 2 hours, the colony has 32 cells. Assuming the growth of the population is exponential, write an equation that represents the population after after t hours.

32. In 2008, some ecologists introduced a new duck population to a local wildlife preserve. The initial duck population was 10 ducks. In 2013 the ecologists noted that the duck population had increased to 35 ducks. If we assume exponential growth, write an equation that represents the population of ducks after t years.

33. A population of bacteria doubles every 6 hours. After 15 hours, the population has 600 cells. How many cells did the population start with?

The remainder of the questions pertain to trigonometry. For extra practice, consult chapter 4 and sections 5.1, 5.2, and 5.5. For questions 34 - 39 compute the exact value of the following expressions.

34. $\sin\left(\frac{21\pi}{4}\right)$

35. $\cos\left(-\frac{\pi}{6}\right)$

36. $\tan(-15^\circ)$

37. $\cos^{-1}\left(\cos\left(-\frac{4\pi}{3}\right)\right)$

38. $\tan\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$

39. $\sin\left(\tan^{-1}\left(-2\right)\right)$

40. If $\cos\theta = -\frac{1}{3}$ and $\pi < \theta < \frac{3\pi}{2}$, what is $\tan\theta$?

41. If $\tan\theta = -4$ what is $\tan(\theta + \frac{\pi}{2})$?

42. Prove that $(\cos x + \sin x)^2 = 1 + \sin(2x)$.

43. Give a formula that expresses $\sin(4\theta)$ in terms of $\sin\theta$ and $\cos\theta$.