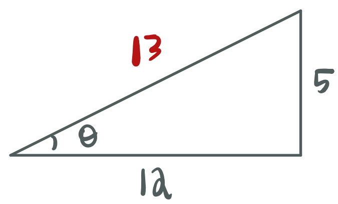
Warm up.

(1)

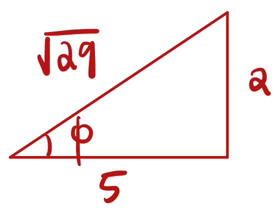


Compute $\cos(2\theta)$ and $\sin(2\theta)$.

$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$= 2\left(\frac{12}{13}\right)^2 - 1$$
$$= 2\cdot\frac{144}{169} - \frac{169}{169} = \frac{119}{169}$$

$$\sin(2\theta) = 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$$
$$= \frac{120}{169}$$

(2) Suppose $\tan\phi = -\frac{2}{5}$ and $90° < \phi < 180°$.
Compute $\tan(2\phi)$ and $\sin(2\phi)$.



$$\cos\phi = \frac{-5}{\sqrt{29}} \qquad \sin\phi = \frac{2}{\sqrt{29}}$$

$$\sin(2\phi) = 2\left(\frac{-5}{\sqrt{29}}\right)\left(\frac{2}{\sqrt{29}}\right) = \frac{-20}{29}$$

$$\tan(2\phi) = \frac{2\left(-\frac{2}{5}\right)}{1-\left(-\frac{2}{5}\right)^2} = \frac{-\frac{4}{5}}{\frac{21}{25}} = -\frac{4}{\cancel{5}}\cdot\frac{\overset{5}{\cancel{25}}}{21}$$
$$= \frac{-20}{21}$$

Last class we derrived the double-angle formulas:

- $\cos(2\theta) = 2\cos^2\theta - 1$
  $= 1 - 2\sin^2\theta$
  $= \cos^2\theta - \sin^2\theta$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Now we want to use these to derive half angle formulas.

$\cos(2\theta) = 2\cos^2\theta - 1$ $\qquad\qquad \theta = \frac{1}{2}\phi$

$$\cos\left(2\cdot\tfrac{1}{2}\phi\right) = 2\underbrace{\cos^2\left(\tfrac{1}{2}\phi\right)}_{\text{isolate } \cos\left(\frac{1}{2}\phi\right)} - 1$$

$$\cos(\phi) = 2\cos^2\left(\tfrac{1}{2}\phi\right) - 1$$

$$1 + \cos\phi = 2\cos^2\left(\tfrac{1}{2}\phi\right)$$

$$\frac{1+\cos\phi}{2} = \cos^2\left(\tfrac{1}{2}\phi\right)$$

$$\pm\sqrt{\frac{1+\cos\phi}{2}} = \cos\left(\tfrac{1}{2}\phi\right)$$

WHAT SIGN DO WE PICK??

↳ depends on what quadrant $\frac{1}{2}\phi$ lives in

(3) Compute $\cos\left(\frac{\pi}{12}\right)$.

$$\cos\left(\frac{\pi}{12}\right) = \sqrt{\frac{\cos\left(\frac{\pi}{6}\right)+1}{2}} = \sqrt{\frac{\sqrt{3}+2}{4}}$$

$$= \frac{\sqrt{\sqrt{3}+2}}{2}$$

(4) Compute $\cos(112.5^\circ)$

$$\cos(112.5^\circ) = \pm\sqrt{\frac{\cos(225^\circ)+1}{2}} = \pm\sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \sqrt{2-\sqrt{2}}$$

(6) Fill in the table. Assume $0 \le \phi < 360°$.

Quadrant of ϕ	Quadrant of $\frac{1}{2}\phi$
1	
2	
3	
4	

(7) Does the above hold if $\phi > 360°$?

We can use $\cos(2\theta) = 1 - 2\sin^2\theta$ to compute a sin half-angle formula as well,

$$\sin\left(\tfrac{1}{2}\phi\right) = \pm\sqrt{\frac{1-\cos\phi}{2}}$$

again use the quadrant of the angle to compute the sign

$$\tan\left(\tfrac{1}{2}\phi\right) = \frac{\sin\left(\frac{1}{2}\phi\right)}{\cos\left(\frac{1}{2}\phi\right)}$$

$$= \frac{\pm\sqrt{\frac{1-\cos\phi}{\cancel{2}}}}{\pm\sqrt{\frac{1+\cos\phi}{\cancel{2}}}}$$

$$= \pm\frac{\sqrt{1-\cos\phi}}{\sqrt{1+\cos\phi}} \cdot \frac{\sqrt{1-\cos\phi}}{\sqrt{1-\cos\phi}}$$

$$= \pm\frac{1-\cos\phi}{\sqrt{\underbrace{1-\cos^2\phi}_{=\sin^2\phi}}} = \pm\frac{1-\cos\phi}{\sin\phi}$$

What about the sign?

$1-\cos\phi > 0$

$\Downarrow$

$\text{sign}\sin\phi = \text{sign}\tan\left(\tfrac{1}{2}\phi\right)$

Think about why this holds!

$$\tan\left(\frac{1}{2}\phi\right) = \frac{1 - \cos\phi}{\sin o}$$