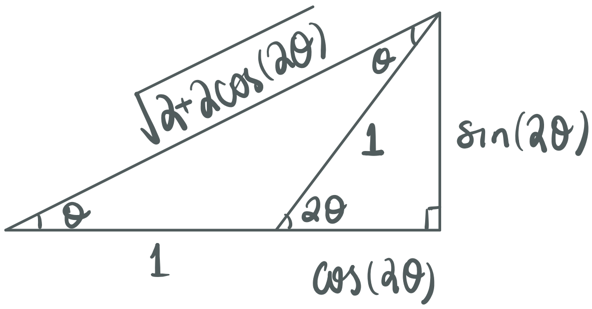

$$\cos\theta = \frac{1+\cos(2\theta)}{\sqrt{2+2\cos(2\theta)}}$$

$$\cos^2\theta = \frac{(1+\cos(2\theta))^2}{2+2\cos(2\theta)}$$

$$= \frac{(1+\cos(2\theta))^2}{2(1+\cos(2\theta))}$$

$$\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$$

$$2\cos^2\theta = 1+\cos(2\theta)$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

(1) Can you write an expression for $\cos(2\theta)$ in terms of $\sin\theta$?

use the fact that $\cos^2\theta + \sin^2\theta = 1$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$= 2(1-\sin^2\theta) - 1$$

$$= 1 - 2\sin^2\theta$$

(2) Challenge: can you rewrite the expression so it's in terms of sin & cos but has no constant?

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$= 2\cos^2\theta - (\cos^2\theta + \sin^2\theta)$$

$$= \cos^2\theta - \sin^2\theta$$

<u>Note.</u> We only showed this for small angles, but it's true for all angles.

Now let's find an expression for $\sin(2\theta)$.

$$\sin\theta = \frac{\sin(2\theta)}{\sqrt{2+2\cos(2\theta)}}$$

$$= \frac{\sin(2\theta)}{\sqrt{2+2(2\cos^2\theta-1)}}$$

$$\sin\theta = \frac{\sin(2\theta)}{\sqrt{2+4\cos^2\theta-2}}$$

$$\sin\theta = \frac{\sin(2\theta)}{\sqrt{4\cos^2\theta}}$$

$$\sin\theta = \frac{\sin(2\theta)}{2\cos\theta}$$

$$2\sin\theta\cos\theta = \sin(2\theta)$$

(3) Can you find a formula for $\tan(2\theta)$?

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

(4) Compute $\sin(2\theta)$ & $\cos(2\theta)$