

Riddle #4: Try to Solve This Pickleball Puzzle

(Difficulty: **HARD**)

Kenny, Abby, and Ned got together for a round-robin pickleball tournament, where, as usual, the winner stays on after each game to play the person who sat out that game. At the end of their pickleball afternoon, Abby is exhausted, having played the last seven straight games. Kenny, who is less winded, tallies up the games played:

Kenny played **eight** games

Abby played **12** games

Ned played **14** games

Who won the fourth game against whom?

Hint

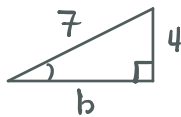
How many total games were played?

Ned beat Kenny

Warm up problems.

$$(1) \quad \sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$$

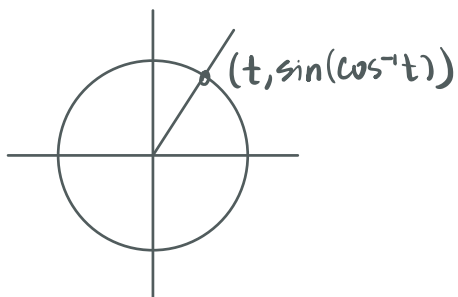
$$(2) \quad \cos\left(\sin^{-1}\left(\frac{4}{7}\right)\right) = \frac{\sqrt{33}}{7}$$


$$b^2 = 7^2 - 4^2 \Rightarrow b = \sqrt{33}$$

$$(3) \quad \tan(\tan^{-1}(e)) = e$$

From Wednesday.

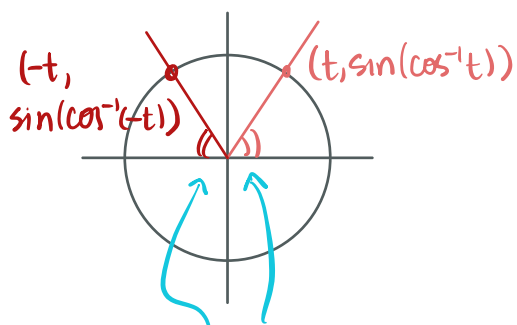
Let $0 < t < 1$, draw $\cos^{-1}t$ in the proper quadrant,
this is an angle!



Now let's draw the angle representing $\cos^{-1}(-t)$...

a few facts:

- should be between $[0, \pi]$
- should intersect the unit circle where $x = -t$



two angles denote are the same!

So we get the relation,

$$\cos^{-1}(-t) = \pi - \cos^{-1}t$$

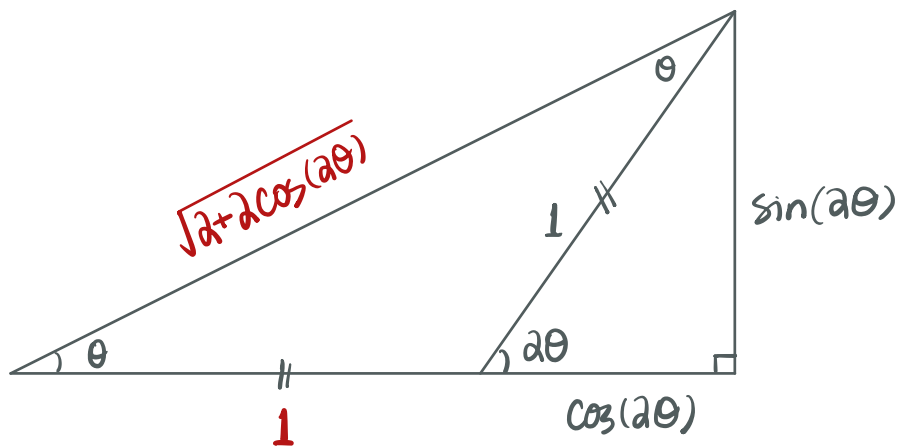
We also have relationships for the arcsine and arctangent...

$$\sin^{-1}(-t) = -\sin^{-1}t$$

$$\tan^{-1}(-t) = -\tan^{-1}t$$

(4) Assuming $0 < t < 1$ draw two pictures (like the one above) depicting the above two identities.

Let's assume $0 < \theta < \frac{\pi}{2}$,



Question: what's the length of the hypotenuse?

$$\cos \theta = \frac{1 + \cos(2\theta)}{\sqrt{2 + 2\cos(2\theta)}}$$

$$\begin{aligned} \cos^2 \theta &= \frac{(1 + \cos(2\theta))^2}{2 + 2\cos(2\theta)} = \frac{(1 + \cos(2\theta))^2}{2(1 + \cos(2\theta))} \\ &= \frac{1 + \cos(2\theta)}{2} \end{aligned}$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos(2\theta))$$

$$A: \mathbb{R} \quad N: 5 \quad K: 4$$

17) A N

16) A K

15) A N

14) A K

13) A N

12) A K

11) A N

10) N K

9) N A

8) K N

7) A N

6) K N

5) A N Ned

4) K N

3) A N

2) K N

1) A N