Recall the following useful table...

	Domain	Range
$\sin^{-1} x$	$[-1,1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1,1]$	$[0, \pi]$
$\tan^{-1} x$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$

Group questions.

(1) Compute $\tan(\tan^{-1}(\frac{1}{\sqrt{3}}))$.

$\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$

(2) Compute $\sin(\sin^{-1}(\frac{-1}{\sqrt{2}}))$

$\sin(\frac{-\pi}{4}) = \frac{-1}{\sqrt{2}}$

(3) Compute $\cos(\cos^{-1}(\frac{1}{5}))$.

$\frac{1}{5}$

(4) Compute $\cos^{-1}\left(\cos\frac{\pi}{2}\right)$.

$\cos^{-1}(0) = \frac{\pi}{2}$

(5) Compute $\tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right)$.

$\tan^{-1}(1) = \frac{\pi}{4}$

(6) Compute $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$.

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{-\pi}{3}$

(7) Compute $\cos^{-1}\left(\cos\left(\frac{-\pi}{6}\right)\right)$.

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

We have the following identities.

$\cos(\cos^{-1} t) = t$ for all $t \in [-1, 1]$

$\sin(\sin^{-1} t) = t$ for all $t \in [-1, 1]$

$\tan(\tan^{-1} t) = t$ for all t.

$\cos^{-1}(\cos\theta) = \theta$ for all $\theta \in [0, \pi]$

$\sin^{-1}(\sin\theta) = \theta$ for all $\theta \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

$\tan^{-1}(\tan\theta) = \theta$ for all $\theta \in (\frac{-\pi}{2}, \frac{\pi}{2})$

More group work.

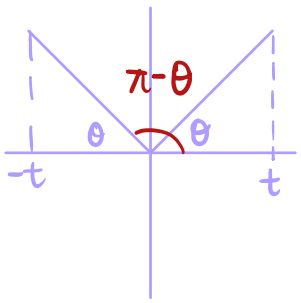
Important: think of $\sin^{-1} t$ as an ANGLE which I can draw in the plane (same for $\cos^{-1} t$ & $\tan^{-1} t$)

18) Suppose $0 < t < 1$. If $\cos^{-1} t = \theta$, what quadrant does θ lie in?

First quadrant

(9) Compute a formula for $\cos^{-1}(-t)$ in terms of θ.

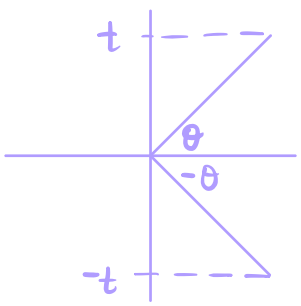
Hint: draw the angle that corresponds to $\cos^{-1}(-t)$ on the unit circle



$\cos^{-1}(-t) = \pi - \theta$

$= \pi - \cos^{-1}(t)$

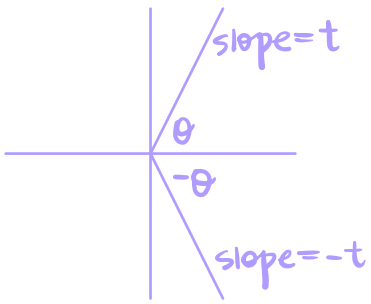
(10) Suppose $0 < t < 1$. Draw the angle that represents $\sin^{-1}t$ and $\sin^{-1}(-t)$.



(11) Can you find a relationship between $\sin^{-1}t$ and $\sin^{-1}(-t)$ using the picture above?

$\sin^{-1}(-t) = -\theta$ & $\sin^{-1}t = \theta \Rightarrow \sin^{-1}(-t) = -\sin^{-1}t$

(12) Do the same for $t>0$ and the angles $\tan^{-1}t$ and $\tan^{-1}(-t)$.



$\tan^{-1}(-t) = -\tan^{-1}(t)$

Summary.

$\cos^{-1}(-t) = \pi - \cos^{-1}t$

$\sin^{-1}(-t) = -\sin^{-1}t$

$\tan^{-1}(-t) = -\tan^{-1}t$