1) Fill out the following table...

	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\mathbb{R}$ except $x = (\frac{\pi}{2} + n)$ where $n \in \mathbb{Z}$	$\mathbb{R}$

2) Sketch a graph of the above functions between -3π and 3π.

3) For each function, find a domain (that includes $x=0$) where the function has an inverse.

Must pick a domain where the function is one-to-one.

The Inverse Trigonometric Functions.

Arcsine, $\sin^{-1}x$

Arccosine, $\cos^{-1}x$

Arctangent, $\tan^{-1}x$

Notation warning!!

$$\sin^{-1}x \neq \frac{1}{\sin x}$$

These are the inverses of their trig counter-parts. This means.

$$\begin{tikzcd}
\text{Angle in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \arrow[r, bend left, "\sin"] & \text{number in } [-1,1] \arrow[l, bend left, "\sin"]
\end{tikzcd}$$

In order to define the inverse, we had to make a choice of a one-to-one domain.

There is a convention of what to pick...

	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$

Ex(1). Compute $\cos^{-1}(1)$.

Reframe the question: for what angle θ such that $\underbrace{\theta \in [0, \pi]}$ is $\cos\theta = 1$.

this ensures
there is only one

$\theta = 0$ (check: $\cos(0) = 1$)

4) $\sin^{-1}(-1)$

Want $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin\theta = -1$

$\boxed{\theta = -\frac{\pi}{2}}$ NOT $\frac{3\pi}{2}$

5) $\tan^{-1}(\sqrt{3})$

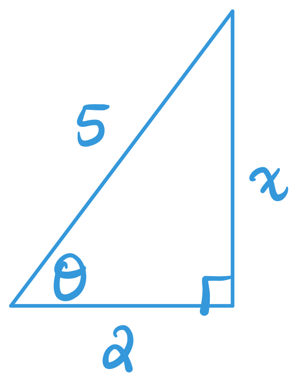
Want $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\tan\theta = \sqrt{3}$

$\boxed{\theta = \frac{\pi}{3}}$

CHALLENGE: $\sin\left(\cos^{-1}\left(\frac{2}{5}\right)\right)$

From the picture $\cos\theta = \frac{2}{5}$

$\Rightarrow \cos^{-1}\left(\frac{2}{5}\right) = \theta$



$x^2 + 2^2 = 5^2$

$x = \sqrt{21}$

$$\sin\left(\cos^{-1}\left(\frac{2}{5}\right)\right) = \sin\theta$$

$$= \frac{\sqrt{21}}{5}$$