(1) (a) $\sec\left(\frac{\pi}{6}\right) = \dfrac{1}{\cos\left(\frac{\pi}{6}\right)}$

$$= \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

(b) $\cot\left(\frac{-3\pi}{4}\right) = \dfrac{1}{\tan\left(\frac{-3\pi}{4}\right)}$

$$= \frac{1}{\tan\left(\frac{5\pi}{4}\right)} = \frac{1}{1} = 1$$

(c) $\csc\left(\frac{-13\pi}{6}\right) = \dfrac{1}{\sin\left(\frac{-13\pi}{6}\right)}$

$$= \frac{1}{\sin\left(\frac{-\pi}{6}\right)} = \frac{1}{-\frac{1}{2}} = -2$$

(d) $\frac{34\pi}{6} = \frac{17\pi}{3}$!!

$$\cot\left(\frac{17\pi}{3}\right) = \cot\left(\frac{11\pi}{3}\right) = \cot\left(\frac{5\pi}{3}\right)$$

$$= \frac{1}{\tan\left(\frac{5\pi}{3}\right)} = \frac{1}{-\sqrt{3}} =$$

(2) $\cot\theta = \dfrac{\csc\theta}{\sec\theta}$

$$\frac{\csc\theta}{\sec\theta} = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \cot\theta \qquad \square$$

(3) $1+\tan^2\theta = \sec^2\theta$

$$1+\tan^2\theta = 1+\frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$= \left(\frac{1}{\cos\theta}\right)^2$$

$$= \sec^2\theta$$

$\square$

4.6 Properties of sin, cos, and tan.

Even/odd properties.

- $\cos\theta$ is even $\Rightarrow$ $\cos(-\theta) = \cos\theta$.
- $\sin\theta$ is odd $\Rightarrow$ $\sin(-\theta) = -\sin\theta$
- $\tan\theta$ is odd $\Rightarrow$ $\tan(-\theta) = -\tan\theta$

Ex(1). Compute $\sin\left(\frac{-\pi}{4}\right)$

$$\sin\left(\frac{-\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$$

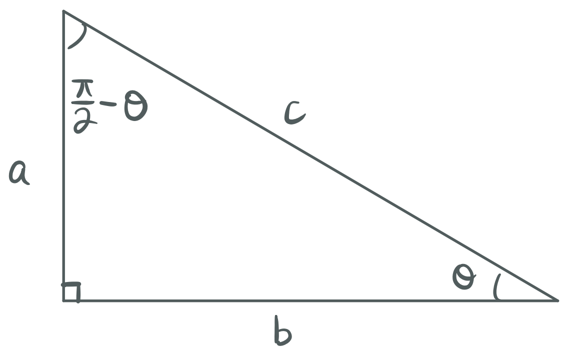
$$= -\frac{1}{\sqrt{2}}$$

Ex(2). Is $\sec\theta$ an even or odd function?

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \sec\theta$$

$\sec(-\theta) = \sec(\theta) \Rightarrow$ sec is even.

Relationship between θ and $\frac{\pi}{2} - \theta$.



$$\sin\theta = \frac{a}{c} = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \frac{b}{c} = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \frac{a}{b} = \frac{1}{\frac{b}{a}} = \frac{1}{\tan\left(\frac{\pi}{2} - \theta\right)}$$

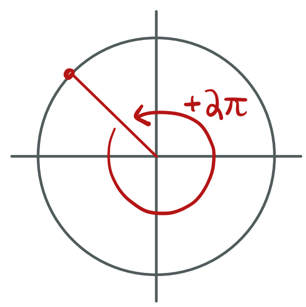
And these hold for ALL angles, not just the ones we see in right triangles.

Trigonometric identities w/ $\frac{\pi}{2} - \theta$.

- $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$
- $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
- $\tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan\theta}$ (*)

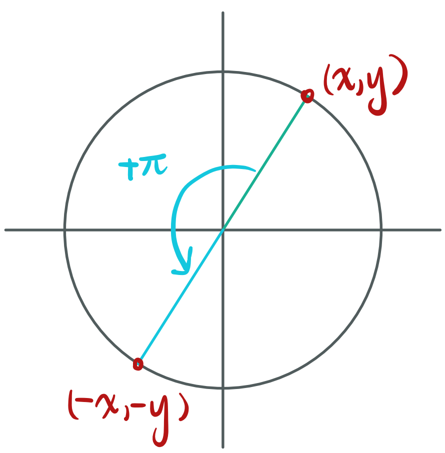
(*) θ cannot be an integer multiple of $\frac{\pi}{2}$.

Trigonometric identities w/ $\theta+2\pi$.



- $\cos(\theta+2\pi) = \cos\theta$
- $\sin(\theta+2\pi) = \sin\theta$
- $\tan(\theta+2\pi) = \tan\theta$

Trigonometric identities w/ $\theta+\pi$.



- $\cos(\theta+\pi) = -\cos\theta$
- $\sin(\theta+\pi) = -\sin\theta$
- $\tan(\theta+\pi) = \tan\theta$

let's prove this one

$$\tan(\theta+\pi) = \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)} = \frac{-\sin\theta}{-\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \square$$

<u>Ex(3).</u> Express $\cos(\theta+5\pi)$ in terms of $\cos\theta$.

$$\cos(\theta+5\pi) = \cos(\theta+3\pi)$$

$$= \cos(\theta+\pi) = -\cos\theta.$$

<u>Ex(4).</u> Express $\sin(7\pi-\theta)$ in terms of $\sin\theta$.

$$\sin(7\pi-\theta) = \sin(-\theta+7\pi)$$

$$= \sin(-\theta+5\pi)$$

$$= \sin(-\theta+3\pi)$$

$$= \sin(-\theta+\pi) = -\sin(-\theta)$$

$$= -(-\sin\theta) = \sin\theta.$$