

More Trigonometric Function (4.4).

Secant. $\sec \theta = \frac{1}{\cos \theta}$

Cosecant. $\csc \theta = \frac{1}{\sin \theta}$

Cotangent. $\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$

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(1) Find an angle with the same unit circle radius that's between 0 and 2π .

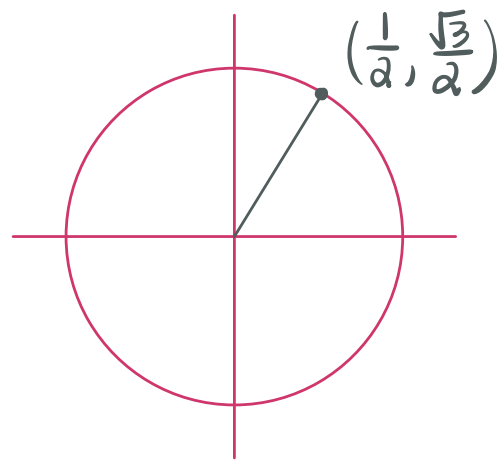
$$-\frac{5\pi}{3} + 2\pi = -\frac{5\pi}{3} + \frac{6\pi}{3} = \frac{\pi}{3}$$

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(2) Identify the point on the unit circle associated to the point.

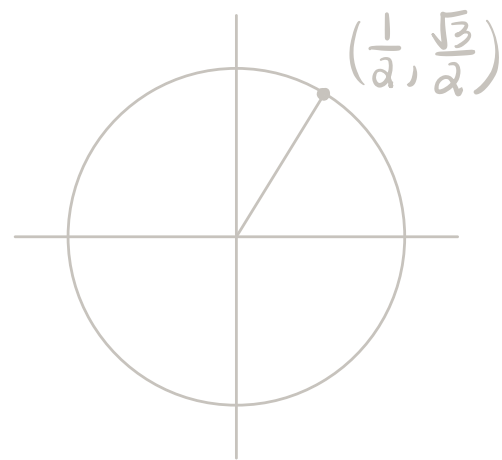


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(3) Compute cosecant.

$$\csc\left(-\frac{5\pi}{3}\right) = \frac{1}{\sin\left(-\frac{5\pi}{3}\right)}$$

$$= \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

Notice, $\csc \theta = \frac{1}{\sin \theta}$ is undefined if $\sin \theta = 0$.

Ex(2). For what angles, $0 \leq \theta < 2\pi$, is $\csc \theta$ undefined?

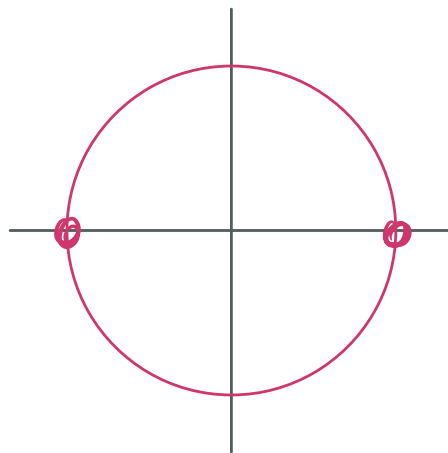
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$$\csc \theta = \text{undef.} \Rightarrow \sin \theta = 0$$

\Updownarrow
 $y = 0$ on
unit circle

$$\theta = 0, \pi.$$



Ex (3). Find ALL angles where the cosecant is undefined.

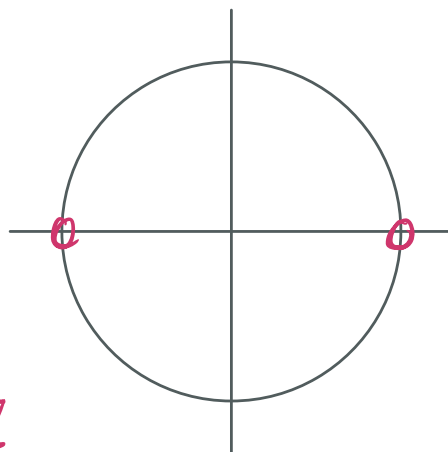
Ex (3). Find ALL angles where the cosecant is undefined.

$$\sin \theta = 0 \Rightarrow \theta = 0, \pi, 2\pi, 3\pi, \dots$$

and also
 $-\pi, -2\pi, \dots$

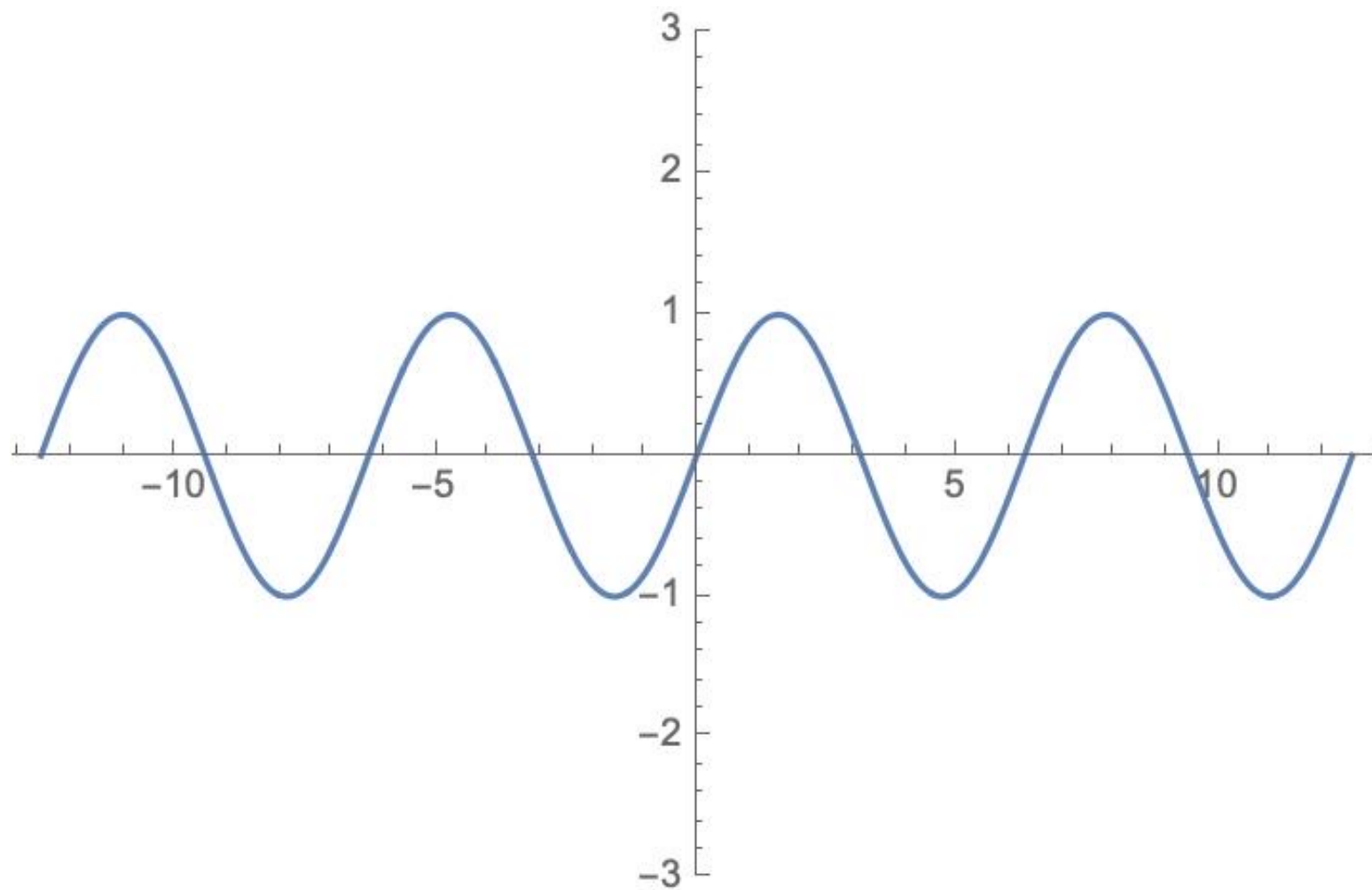
$$\Rightarrow \theta = n\pi \text{ for any } n \in \mathbb{Z}$$

↑
n is an
integer

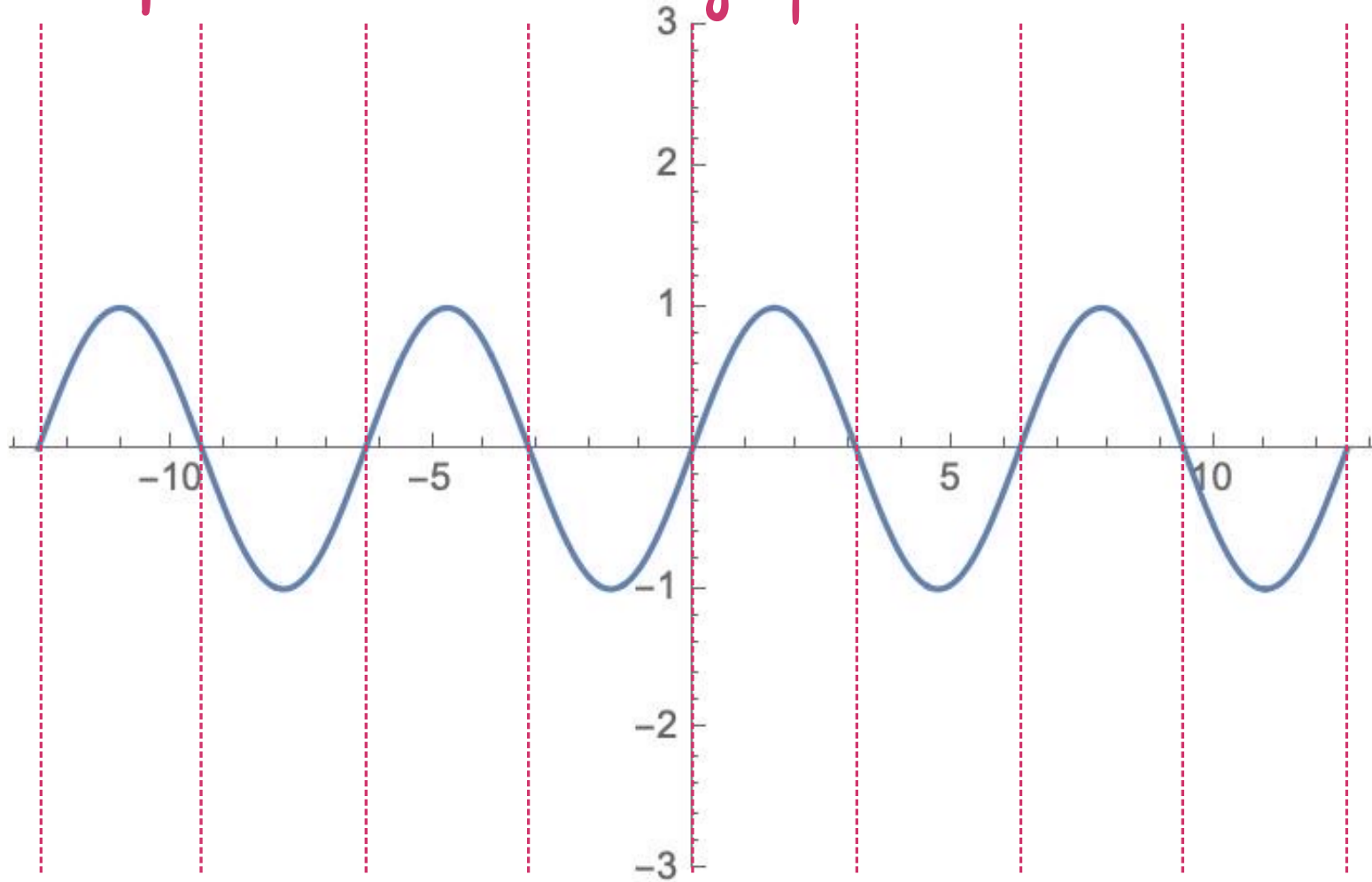


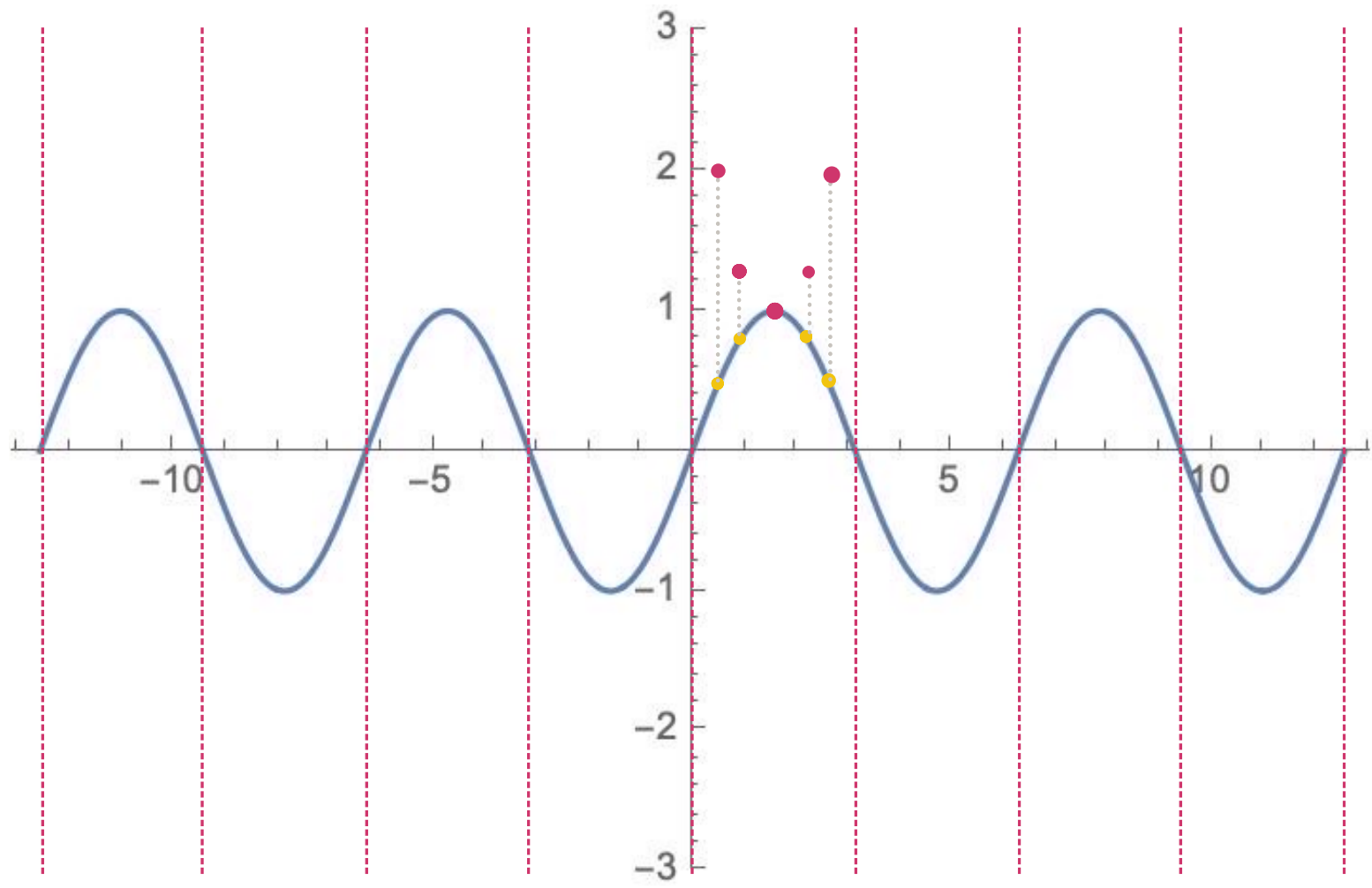
Let's use this and the graph of $f(x) = \sin x$ to graph $f(x) = \csc x$.

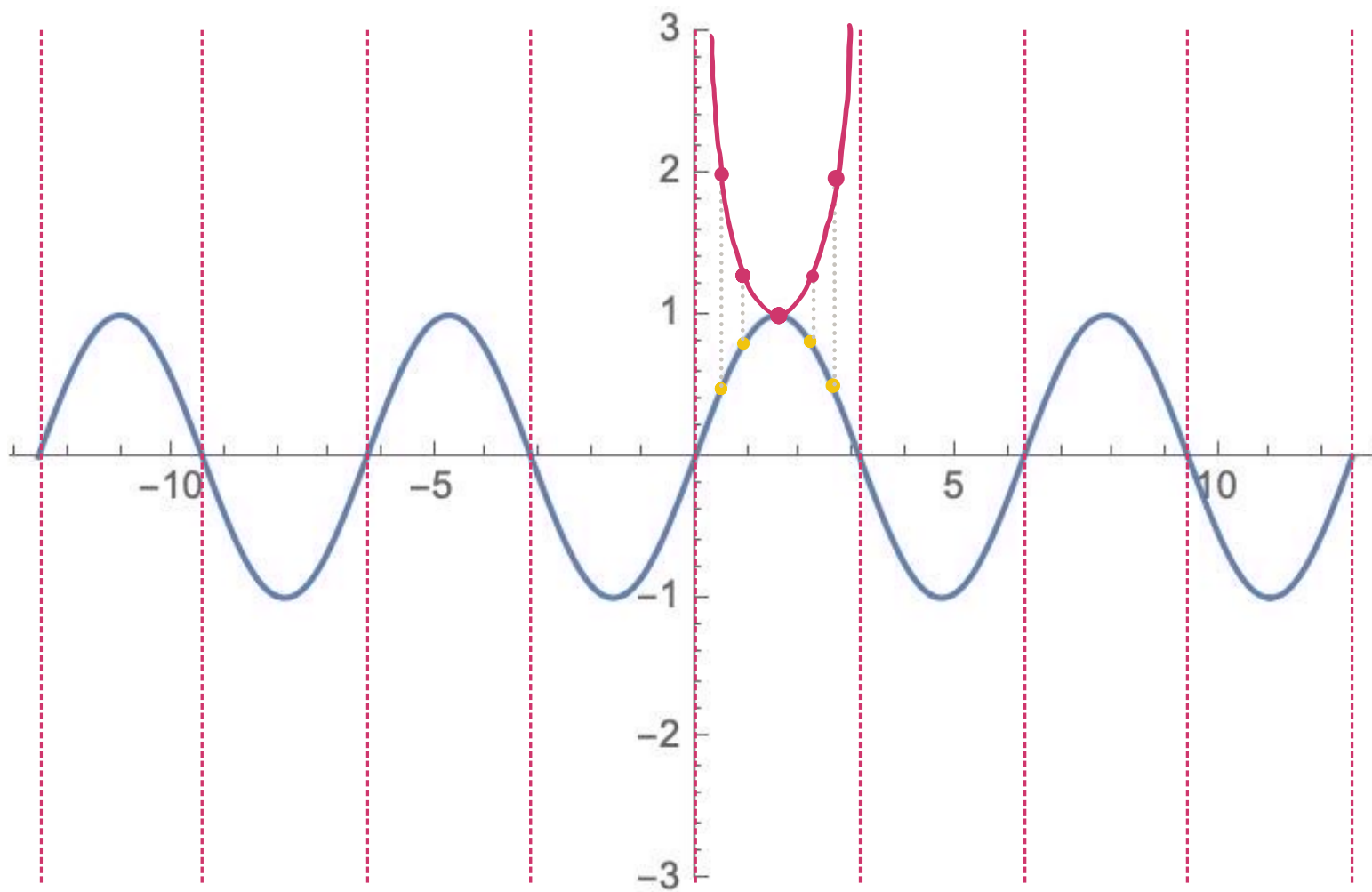
$$f(x) = \sin x$$

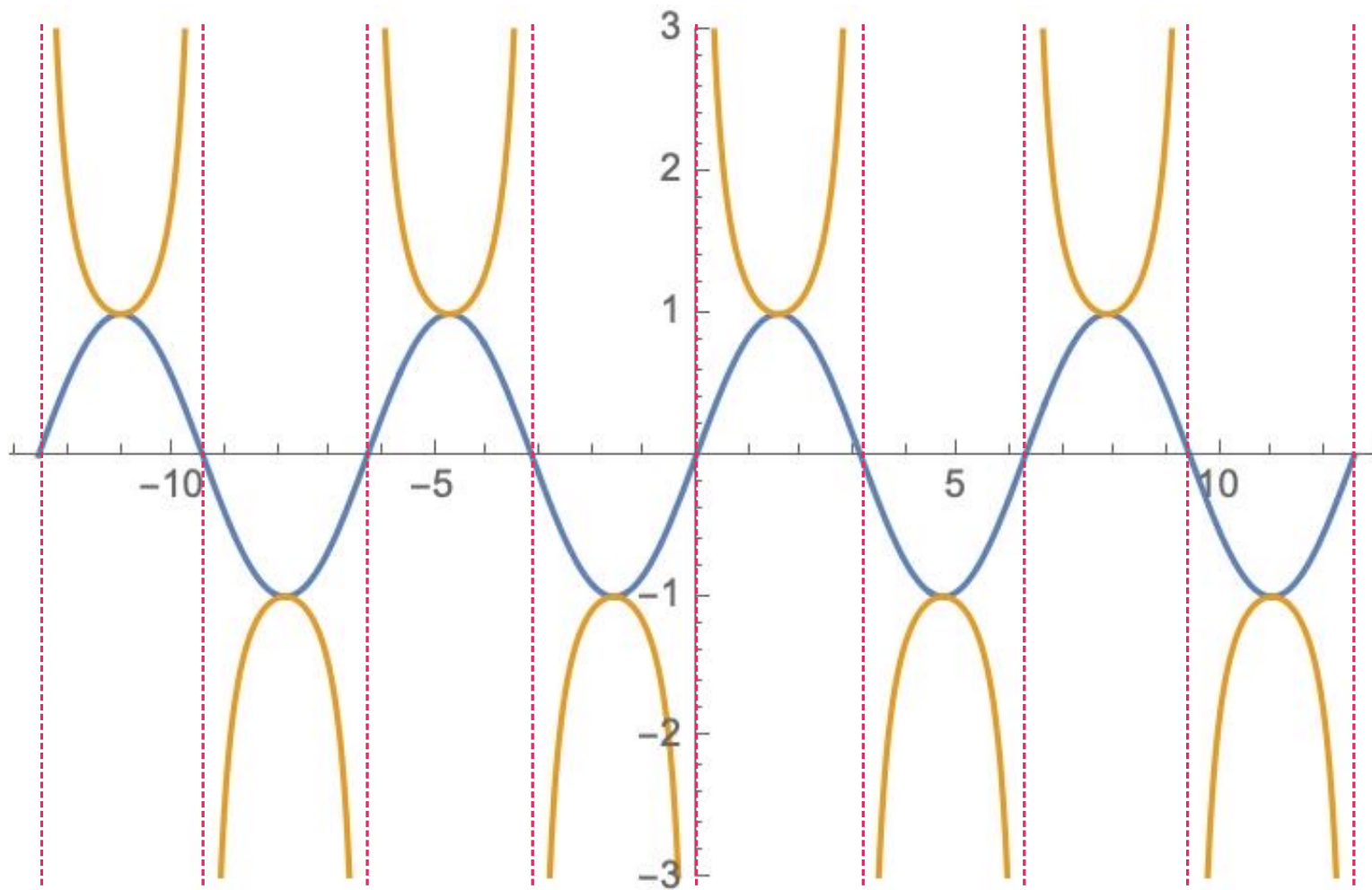


undefined = vertical asymptote









Ex(4). What is the domain and range of $f(x) = \csc x$ (based on the above).

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Domain: all \mathbb{R} except $x = n\pi$ for $n \in \mathbb{Z}$.

Range: $(-\infty, -1] \cup [1, \infty)$

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Ex(5). Compute the domain and range of $f(x) = \sec x$.

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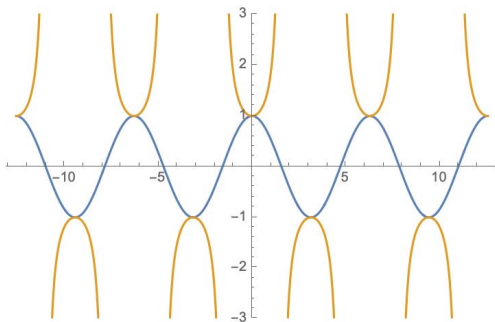
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Ex(5). Compute the domain and range of $f(x) = \sec x$.

$\sec x$ is undefined when $\cos x = 0$.

$\cos x = 0 \Leftrightarrow x = \frac{n\pi}{2}$ for n any odd integer.



Domain: all \mathbb{R} except $x = \frac{n\pi}{2}$
for n any odd integer

Range: $(-\infty, -1] \cup [1, \infty)$

Additional exercises (if time).

(1) Compute the following,

(a) $\sec\left(\frac{\pi}{6}\right)$

(b) $\cot\left(-\frac{3\pi}{4}\right)$

(c) $\csc\left(-\frac{13\pi}{6}\right)$

(d) $\cot\left(\frac{34\pi}{6}\right)$

(2) Show that $\cot\theta = \frac{\csc\theta}{\sec\theta}$.

(3) Prove $1 + \tan^2\theta = \sec^2\theta$. (Ask for a hint!)