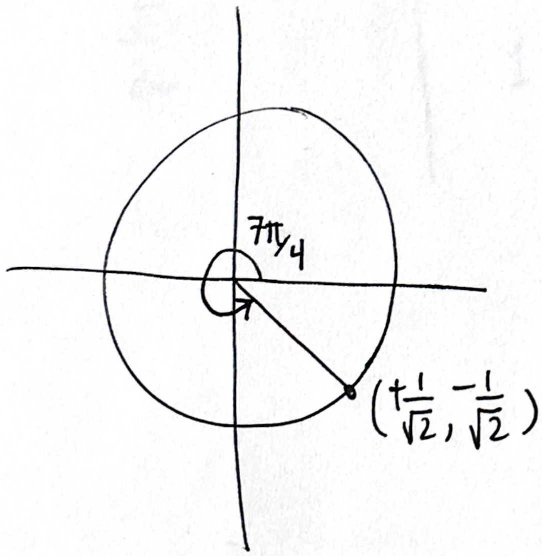
Notes 04/01

①

(*) Start w/ computer drawing of $\sin x$, $\cos x$. Show how to solve the following graphically...

- $\sin x = a$ / $\cos x = b$
- $\sin x = \cos x$ ...
- show that shifting $\sin \to \cos$.

<u>Ex (1).</u> Compute $\cos\left(\frac{7\pi}{4}\right)$ & $\sin\left(\frac{7\pi}{4}\right)$



$$\cos\left(\frac{7\pi}{4}\right) = +\frac{1}{\sqrt{2}}$$

$$\sin\left(\frac{7\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

Recall the following definition of $\tan\theta$ ...

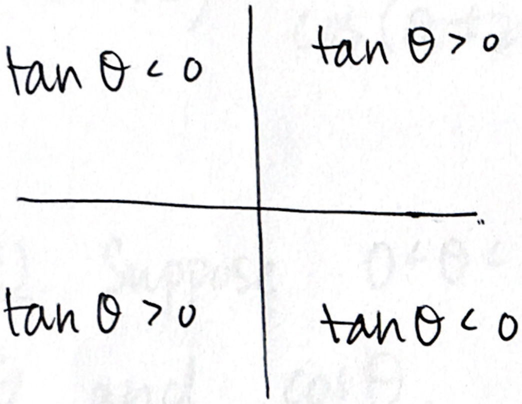
$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$

With this definition in mind we can extend the definition of the tangent to ANY angle.

θ (rads)	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	<u>undefined</u>

$\tan\theta$ is not defined for angles where $\cos\theta = 0$!

The sign of the tangent also depends on the quadrant of the coordinate plane the angle is in.



Ex(2). Compute $\tan\left(\frac{7\pi}{4}\right)$.

In Ex(1) we saw $\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$ & $\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

so

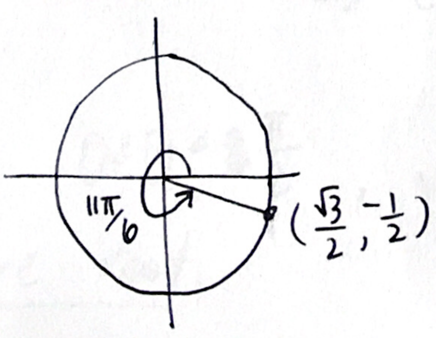
$$\tan\left(\frac{7\pi}{4}\right) = \frac{\sin\left(\frac{7\pi}{4}\right)}{\cos\left(\frac{7\pi}{4}\right)} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1.$$

(*) tan is also 2π-periodic!

Ex(3). Compute $\tan\left(\frac{23\pi}{6}\right)$.

$$\frac{23\pi}{6} - 2\pi = \frac{23\pi}{6} - \frac{12\pi}{6} = \frac{11\pi}{6}$$

now between 0 and 2π



$$\tan\left(\frac{23\pi}{6}\right) = \tan\left(\frac{11\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

Fact. $\tan\theta$ is a 2π-periodic function.

$$\tan(\theta \pm 2\pi) = \frac{\sin(\theta \pm 2\pi)}{\cos(\theta \pm 2\pi)} = \frac{\sin\theta}{\cos\theta} = \tan\theta .$$

Ex(4). Suppose $0 < \theta < \frac{\pi}{2}$ and $\tan\theta = 2$. Compute $\sin\theta$ and $\cos\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow 2 = \frac{\sin\theta}{\cos\theta} \Rightarrow 2\cos\theta = \sin\theta$$

Recall –

$$\sin^2\theta + \cos^2\theta = 1$$

$$(2\cos\theta)^2 + (\cos\theta)^2 = 1$$

$$4\cos^2\theta + \cos^2\theta = 1$$

$$5\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{5}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\sin\theta = \frac{2}{\sqrt{5}}$$

Since $0 < \theta < \frac{\pi}{2}$, we ~~have~~ take the positive square root.

Ex 15). Suppose $\frac{\pi}{2} < \theta < \pi$ and $\tan\theta = -4$. Compute $\sin\theta$ and $\cos\theta$.

$$\frac{\sin\theta}{\cos\theta} = -4 \Rightarrow \sin\theta = -4\cos\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$16\cos^2\theta + \cos^2\theta = 1$$

$$17\cos^2\theta = 1$$

$$\cos\theta = \pm\frac{1}{\sqrt{17}} \leftarrow$$ since θ is in quadrant II, $\cos\theta < 0$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{17}}$$

$$\sin\theta = -4 \cdot \frac{-1}{\sqrt{17}} = \frac{4}{\sqrt{17}}$$