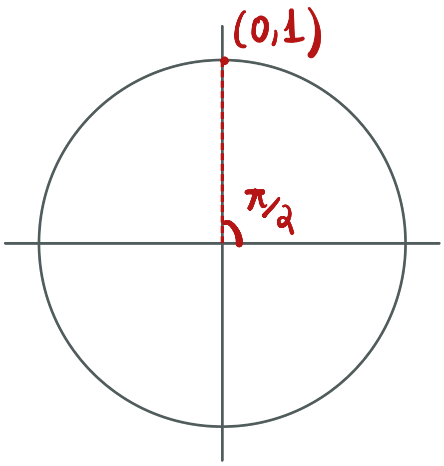
4.3 Sine and Cosine.

(1) Let's talk about sine & cosine for arbitrary angles.

Ex(1). Compute $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$



Remember the pt on the coordinate on the unit circle is $(\cos\theta, \sin\theta)$.

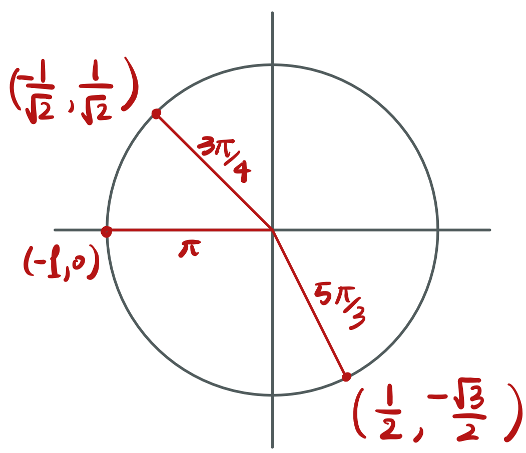
$\Rightarrow \cos\frac{\pi}{2} = 0$ & $\sin\frac{\pi}{2} = 1$

Exercises.

(1) Compute $\cos(\pi)$

(2) Compute $\sin\left(\frac{3\pi}{4}\right)$

(3) Compute $\cos\left(\frac{5\pi}{3}\right)$



We see that the sign of the sin & cosine changes depending on the quadrant the angle lies in.

$\cos\theta < 0$ $\sin\theta > 0$	$\cos\theta > 0$ $\sin\theta > 0$
$\cos\theta < 0$ $\sin\theta < 0$	$\cos\theta > 0$ $\sin\theta < 0$

There's a handful of properties about the sine and cosine we can realize by this unit circle definition...

(1) $\cos^2\theta + \sin^2\theta = 1$

$\sin^2\theta$ means $(\sin\theta)^2$ NOT $\sin(\theta^2)$!!

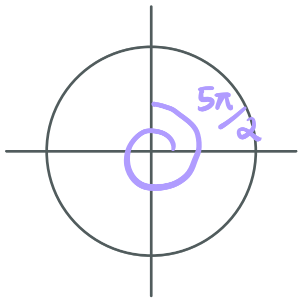
Since pts on the unit circle are of the form $(\cos\theta, \sin\theta)$ AND satisfy $x^2+y^2=1$, the result is immediate.

(2) $-1 \leq \cos\theta \leq 1$, $-1 \leq \sin\theta \leq 1$

Again because the pt $(\cos\theta, \sin\theta)$ lies on the unit circle.

Ex(2). Compute $\sin\frac{5\pi}{2}$ & $\cos\frac{5\pi}{2}$

$\frac{5\pi}{2}$ represents the same pt on the unit circle as $\frac{\pi}{2}$:



$$\frac{5\pi}{2} - 2\pi = \frac{5\pi}{2} - \frac{4\pi}{2}$$
$$= \frac{\pi}{2}$$

$\Rightarrow \cos\frac{5\pi}{2} = 0$, $\sin\frac{5\pi}{2} = 1$

(3) $\cos\theta$ and $\sin\theta$ are 2π-periodic functions.

this means,
$\cos(\theta \pm 2\pi) = \cos\theta$
$\sin(\theta \pm 2\pi) = \sin\theta$

Ex(3). Compute $\sin\left(-\frac{2\pi}{3}\right)$

$$-\frac{2\pi}{3} + 2\pi = -\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{4\pi}{3}$$

$$\sin\left(\frac{-2\pi}{3}\right) = \sin\left(\frac{-2\pi}{3} + 2\pi\right) = \sin\left(\frac{4\pi}{3}\right)$$
$$= \frac{-\sqrt{3}}{2}$$

The Graphs of sin θ & cos θ.