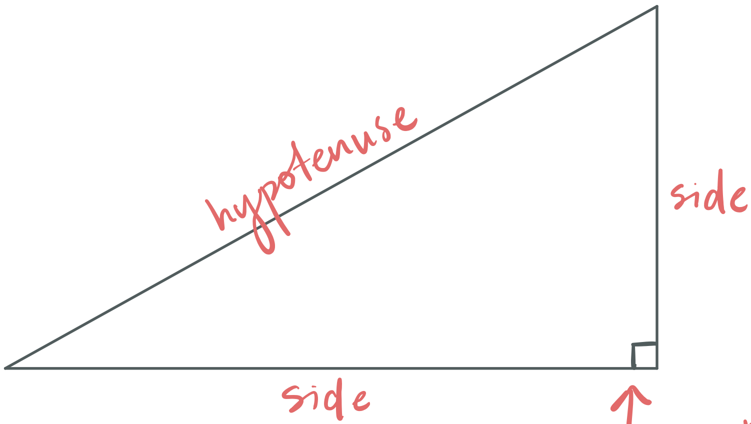
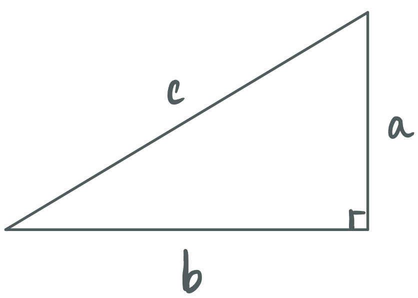
# 4.5 Right Triangles.



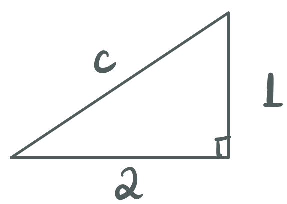

right triangle always has exactly one right angle

## Pythagorean theorem.




$\Rightarrow \quad a^2 + b^2 = c^2$

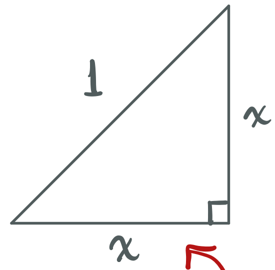
**Ex(1).** Compute $c$,




$$1^2 + 2^2 = c^2$$

$$5 = c^2 \quad \Rightarrow \quad c = \sqrt{5}$$

**Ex (2).** Solve for $x$,




this is a $45°-45°-90°$ triangle

$$x^2 + x^2 = 1$$
$$2x^2 = 1$$
$$x^2 = 1/2 \quad \Rightarrow \quad x = 1/\sqrt{2}$$

## Trigonometric Functions on Right Triangles.

Let's talk about the functions,

(1) Sine (sin)

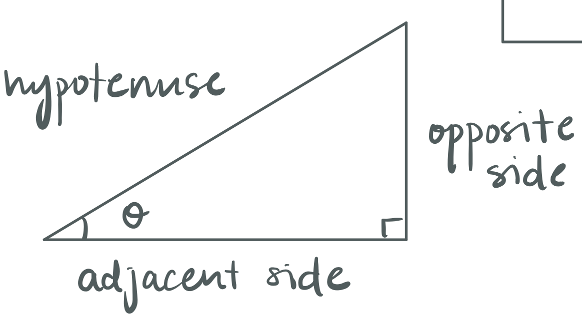
(2) Cosine (cos)

(3) Tangent (tan)

Whose input are angles.

(1) Sine Function.

$$\boxed{\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}}$$

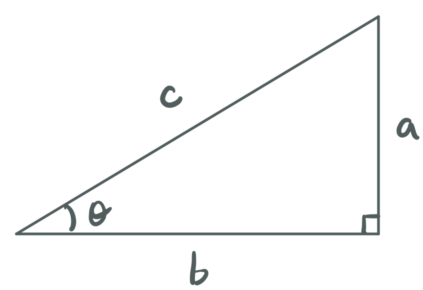


(2) Cosine Function. $\boxed{\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}}$

(3) Tangent Function. $\boxed{\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}}$

<u>Ex (3).</u> Show $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\tan\theta = \dfrac{a}{b}$ (by def.)




$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{a}{c}}{\frac{b}{c}}$$

$$= \frac{a}{c} \cdot \frac{c}{b} = \frac{a}{b}$$

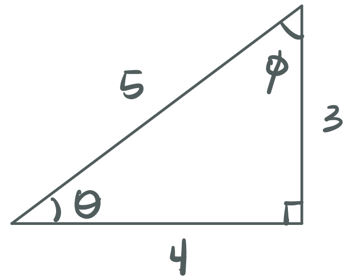
And so $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Exercise. Compute:

(a) $\sin\theta$ (c) $\sin\phi$

(b) $\cos\theta$ (d) $\cos\phi$

(c) $\tan\theta$ (e) $\tan\phi$



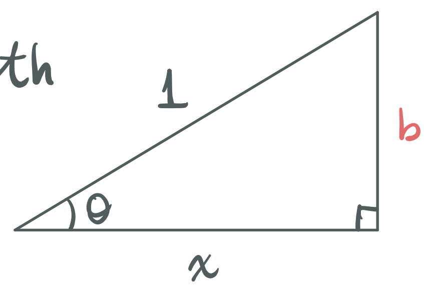

Answers. (a) $\frac{3}{5}$ (d) $\frac{4}{5}$

(b) $\frac{4}{5}$ (e) $\frac{3}{5}$

(c) $\frac{3}{4}$ (f) $\frac{4}{3}$

Ex (4). Compute $\sin\theta$ (in terms of $x$).

Need to compute the length of the opposite side (in terms of $x$).

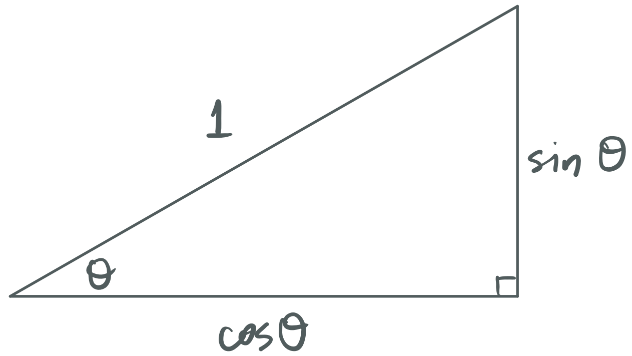



$$b^2 + x^2 = 1^2$$
$$b^2 = 1 - x^2 \Rightarrow b = \sqrt{1-x^2}$$

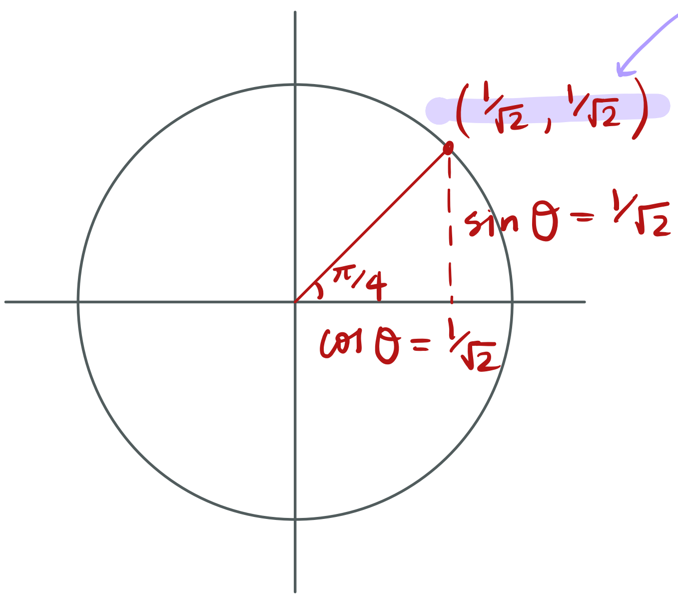
Now we compute $\sin\theta$.

$$\sin\theta = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

If the hypotenuse is 1, then




And this brings us back to the unit circle...




This coord-pair can be thought of as $(\cos\theta, \sin\theta)$

And this will motivate how we extend these functions to any arbitrary angle...