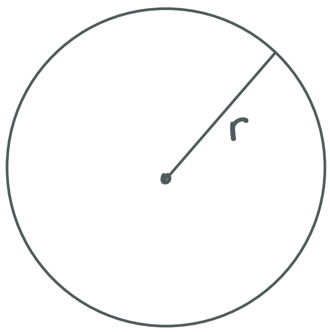
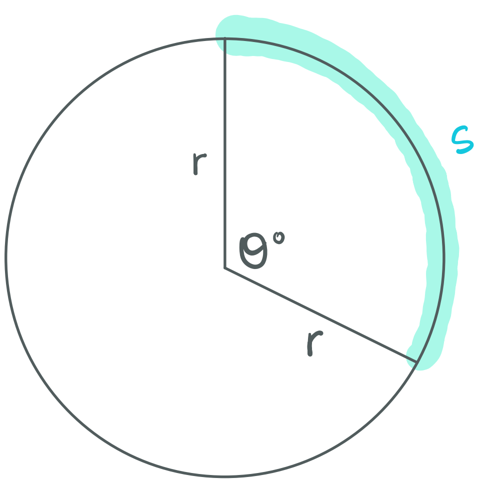
Recall the formula for the circumference of a circle.



$C = 2\pi r$

So the circumference of the **unit circle** is $C = 2\pi(1) = 2\pi$.

Now let's compute the length of a circular arc that corresponds to a degree $\theta°$.



$s = \frac{\pi}{180} \cdot \theta \cdot r$

So if the circle in question is the unit circle, we get

$s = \frac{\pi}{180}\theta$ ← this will lead us to a new way to measure angles

Radians are a unit of measurement for angles. Given an angle in degrees, the angle in radians is equal to the length of the circular arc on the unit circle that corresponds to that angle.

Some common conversions...

$180° \Leftrightarrow \pi$ rads
$90° \Leftrightarrow \frac{\pi}{2}$ rads
$360° \Leftrightarrow 2\pi$ rads

We use the conversion,

$$s \text{ rads} = \frac{\pi}{180} \cdot \theta°$$

<u>Ex (1).</u> Convert 45° to radians

$$s \text{ rads} = \frac{\pi}{180} \cdot 45° = \frac{\pi}{4}$$

To remember how to convert radians to degrees & degrees to radians, it suffices to just remember on equivalence.

π radians = 180°

this is what I like to remember

Ex(2). Convert $\frac{5\pi}{6}$ radians to degrees.

$$\frac{5\pi}{6} \cancel{\text{radians}} \cdot \frac{180^\circ}{\pi \cancel{\text{rads}}} = \frac{5\cancel{\pi} \cdot \overset{30}{\cancel{180}}^\circ}{\cancel{6}\cancel{\pi}}$$

$$= 150^\circ$$

Some exercises.

(1) Convert the following degrees to radians.

(a) 30°
(b) 270°
(c) -180°
(d) 540°
(e) 135°

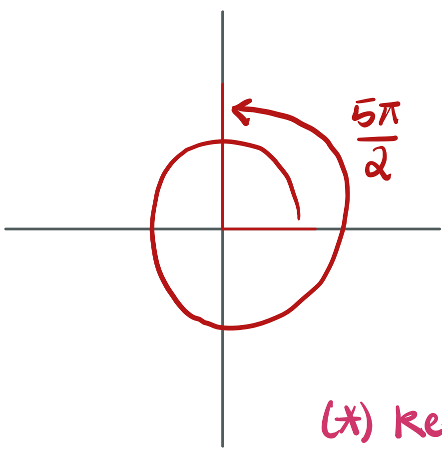
(2) Convert the following radians to degrees.

(a) $\pi/6$
(b) 5π
(c) $-3\pi/4$
(d) $9\pi/10$
(e) $-17\pi/6$

Degrees/radians are repetitive! However ANY angle can be expressed as...

- A degree between $0°$ and $360°$
- A radian between 0 and 2π

Ex(3). Express $\frac{5\pi}{2}$ radians as an angle between 0 and 2π radians



We went around in a circle once, so let's subtract off 2π

$$\frac{5\pi}{2} - 2\pi = \frac{\pi}{2} \text{ radians}$$

(*) Keep subtracting 2π until the angle lies within 0 to 2π.

Ex(4). Express $\frac{19\pi}{4}$ radians as an angle between 0 and 2π radians

$$\frac{19\pi}{4} - 2\pi = \frac{11\pi}{4} - 2\pi = \frac{3\pi}{4} \text{ rads}$$

still greater than 2π

Ex (5). Express $-\frac{5\pi}{6}$ radians as an angle between 0 and 2π radians.

When the angle is negative, we should add 2π until it falls between 0 and 2π.

$$-\frac{5\pi}{6} + 2\pi = \frac{7\pi}{6} \text{ radians}$$