From last class ...

(Pop1) $p_0 = 1000$, $r = 0.1$ per hour

(Pop2) $p_0 = 100$, $r = 0.15$ per hour

Question: When is (Pop2) $\geq$ (Pop1) ?

$p_1(t) = 1000\,e^{0.1t}$ $\qquad$ $p_2(t) = 100\,e^{0.15t}$

Let's compute when they are equal ...

$$1000\,e^{0.1t} = 10\,e^{0.15t}$$

$$\frac{1000}{100} = \frac{e^{0.15t}}{e^{0.1t}} \quad \Rightarrow \quad 10 = e^{0.05t}$$

$$\ln 10 = 0.05t$$

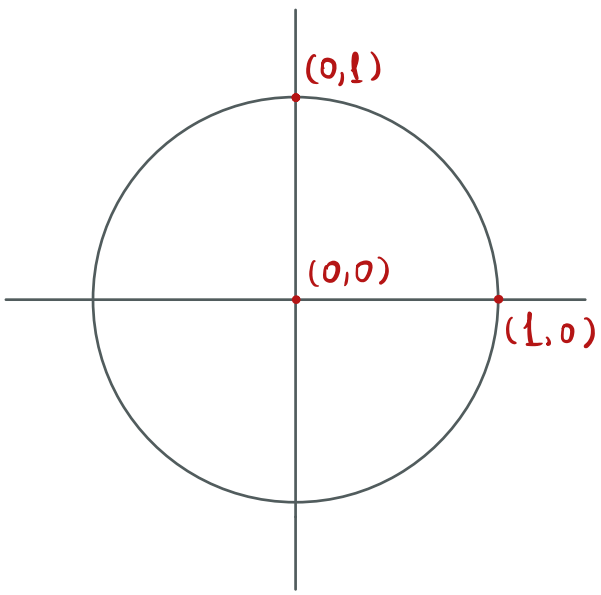
$$\frac{\ln 10}{0.05} = t \sim 46 \text{ hrs.}$$

So (Pop2) $\geq$ (Pop1) when $t \geq \frac{\ln 10}{0.05}$.

The Unit Circle. (4.1)

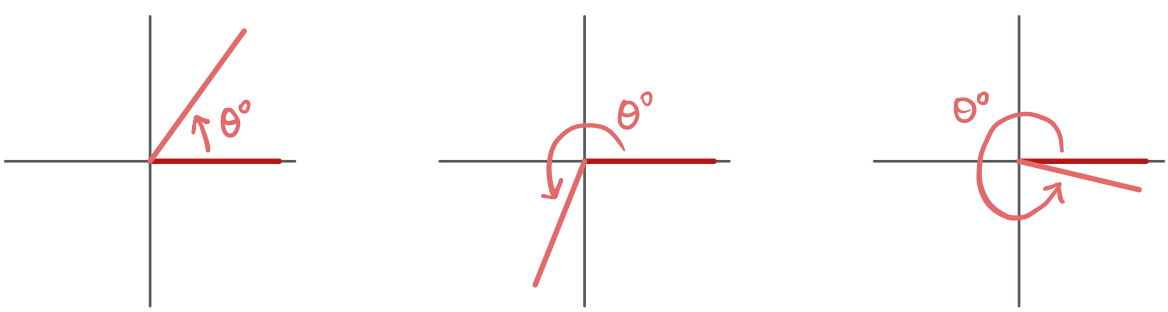
Q What is the unit circle?

A Circle with radius $r=1$ and center $(0,0)$.



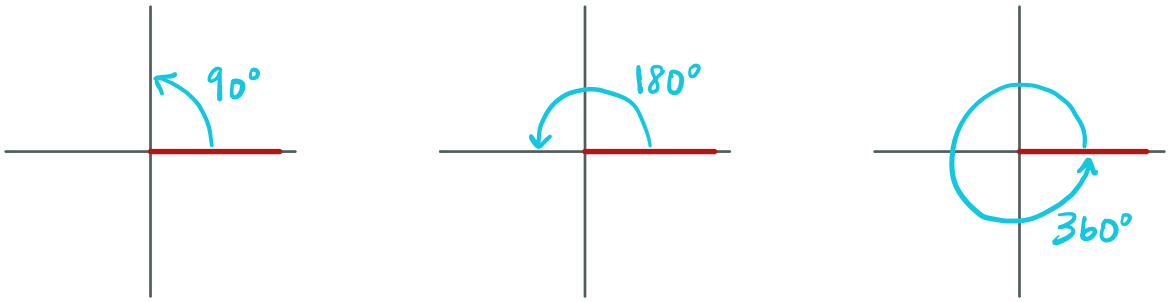
The unit circle is very useful in trigonometry!

Angles in the coordinate plane.



We measure degrees by rotating CCW from the positive horizontal axis.

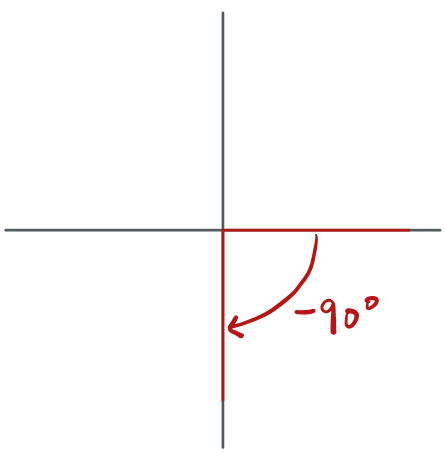
↑ counter clockwise



What about negative angles? (clockwise)

↳ rotate CW |θ|°.

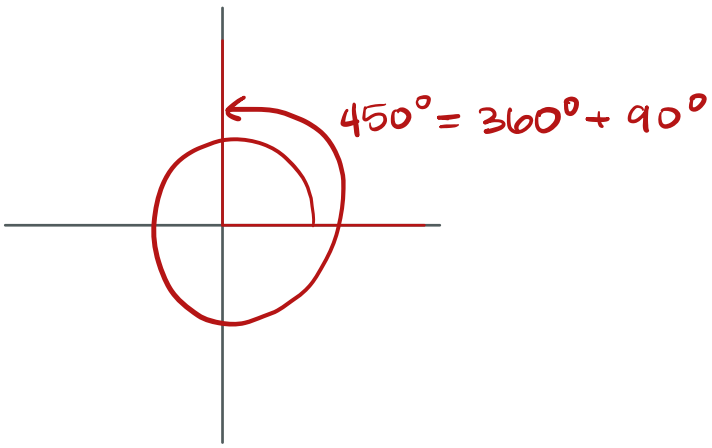
EX(1) Draw the radius that represents −90°.



Notice that the radius we draw for −90° is the same as the radius for 270°.

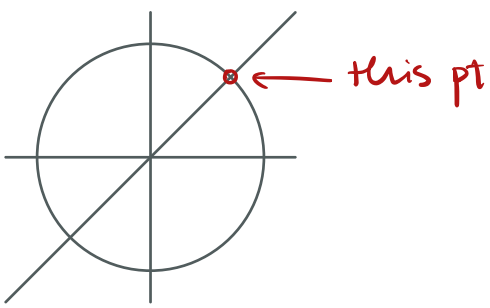
What if the angle is larger than 360°?
↳ just loops around

Ex(2). Draw the radius that represents 450°



Where some radiuses intersect the unit circle.

Ex(3). Where does the line $y=x$ intersect the unit circle in the positive quadrant?



$x^2 + y^2 = 1$ & $y = x$

$x^2 + x^2 = 1$

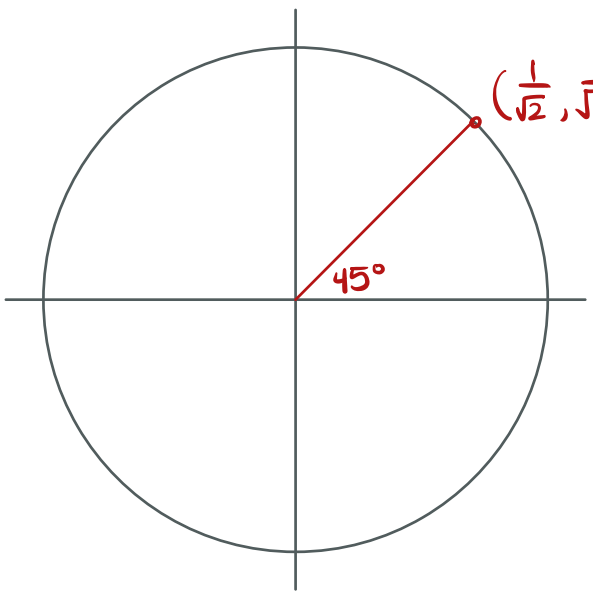
$2x^2 = 1$

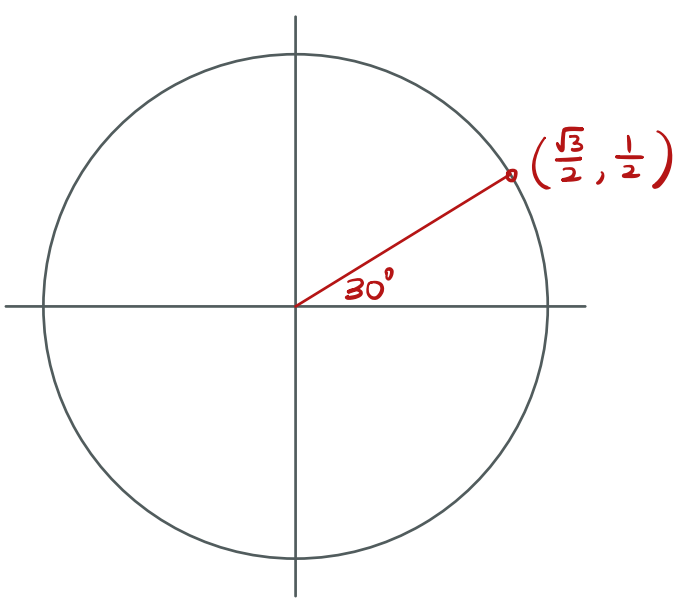
$x^2 = \frac{1}{2}$

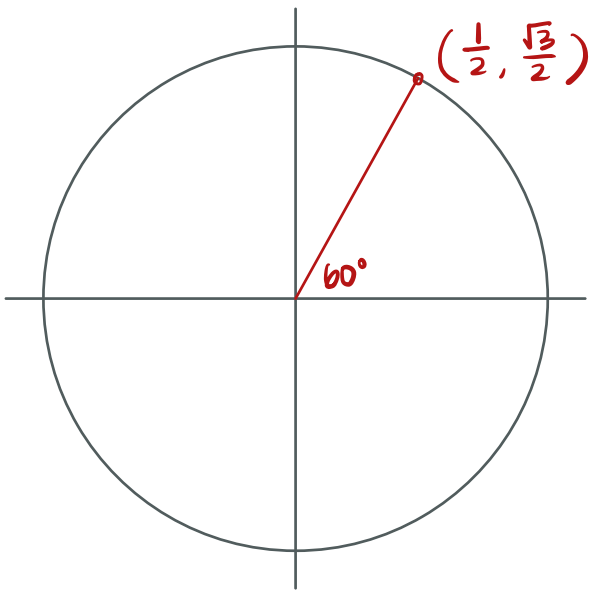
$x = \frac{1}{\sqrt{2}} = y$

Notice that the angle created by the line $y=x$ is $45°$.

Let's see some more notable intersection pts and their angles...







Ex(4). Find where the radius with angle $-60°$ intersects the unit circle.