A function with **exponential growth** has the form,

$$f(x) = c \cdot b^x$$

where $c > 0$, $b > 1$.

Ex (1). $f(x) = 3 \cdot 2^x$

x	$f(x)$
1	6
2	12
3	24
4	48
5	96

← the function gets large VERY fast

Ex (2). $f(x) = 4 \cdot 3^{2x}$

$$\begin{aligned} f(x) &= 4 \cdot 3^{2x} \\ &= 4 \cdot (3^2)^x \\ &= 4 \cdot 9^x \end{aligned}$$

We get **exponential decay** when $0 < b < 1$.

decreases really fast

Ex(3). Suppose f is a function modeling exponential growth, $f(1)=12$ and $f(3)=108$. Compute a formula for f.

$f(x)=c\cdot b^x$

$12=c\cdot b \rightarrow c=\frac{12}{b}$ (plug this into the 2nd equation)

$108=c\cdot b^3$

$108=\frac{12}{b}b^3=12b^2$

$9=b^2 \Rightarrow b=3 \quad (b>1)$

$12=3c \Rightarrow c=4$

$f(x)=4\cdot 3^x$

Exponential growth is a great way to model populations.

$p(t)=p_0 2^{(t-t_0)/d}$

$p_0 =$ initial population
$t_0 =$ initial time
$d =$ doubling time

Ex(4). Suppose a colony of bacteria has 25 cells. The number of bacteria double every 4 hrs. How many cells will there be after 3 days?

$p_0 = 25$
$d = 4$ hrs
$t_0 = 0$ hrs
$t = 3$ days $= 72$ hrs

$$p(72) = 25 \cdot 2^{72/4}$$
$$= 25 \cdot 2^{18}$$
$$= 6553600$$

Exercise. A species of animal doubles its population every 20 days. If the initial population is 10 animals, how long until there are 500?

Previously we discussed **compounding interest**. Moreover, I hope our conversation about e motivated that interest can be **compounded continuously**.

$f(t) = Pe^{rt}$

r= annual interest rate
P= initial amount
t= time (yrs)

<u>Ex(5).</u> Suppose I place \$100 in a bank account that compounds continuously with a 1% interest rate. How much money will be in the account after 5 years?

$f(t) = 100e^{0.01t}$
$f(5) \sim 105.13$

Suppose it was only compounded monthly...

$100\left(1+\frac{0.01}{12}\right)^{60} \sim 105.12$

How many years will it take to double the money in the account (compounded cont.).

$$200 = 100e^{0.01t}$$
$$2 = e^{0.01t}$$
$$\ln 2 = 0.01t$$
$$\frac{\ln 2}{0.01} = t \sim 69.3 \text{ years}$$

We can also consider **continuous growth rates**

$$p(t) = p_0 e^{rt}$$

p_0 = initial size
t = change in time
r = growth rate per unit time