

OUTLINE:

- (1) review percents/taxes.
- (2) compound interest
- (3) Numberphile video (~10 mins)
- (4) e and \ln .

Ex (1) Suppose I buy a \$40 shirt in Walnut Creek, where sales tax is 8.75%. What do I pay in total?

$$40(1+0.0875) = 40(1.0875) = \$43.50$$

Remember: if I'm given a percent, convert to a decimal/fraction before doing math.

$$x\% = \frac{x}{100}$$

Sales tax: $\text{cost} \times (1 + \text{sales tax})$

↑
total
as a decimal

Ex (2) I buy two hanks of yarn each priced at \$18. At the register I pay \$39.24. How much was sales tax?

$$36 \left(1 + \frac{x}{100}\right) = 39.24$$

$$1 + \frac{x}{100} = 1.09 \Rightarrow x = 9\%$$

Ex (3). Suppose the bank has an annual interest rate of 5%. The bank pays interest once per year. If I deposit \$2500 into the bank, how much will I have in a year?

$$2500(1 + 0.05) = \$2625$$

How much will I have after 5 years?

$$\begin{aligned} & 2500(1+0.05)(1+0.05)(1+0.05)(1+0.05)(1+0.05) \\ & = 2500(1.05)^5 \\ & = 3190.70 \text{ (ish)} \end{aligned}$$

For the above, we would say that interest is compounded once per year at an annual interest rate of 5%.

If an initial amount P is compounded once per year at an annual interest rate r , then after t years the amount is,

$$P(1+r)^t$$

Interest can be compounded more than once a year!

Ex (4). Let's suppose I deposit \$2500 into an account that has a 5% annual interest rate compounded twice annually. How much do I have after 1 year?

$$2500 \left(1 + \frac{0.05}{2}\right) \left(1 + \frac{0.05}{2}\right) = \$2626.56$$

After 5 years?

$$2500(1+0.025)^{10} = \$3200.21$$

The equation, $P(1 + \frac{r}{n})^{nt}$

r = annual interest rate

n = how many times
compounded / yr

t = years

P = initial amount

EX(5). What if it was compounded daily for a (non-leap) year?

$$\$2500 \left(1 + \frac{0.05}{365}\right)^{365} = \$2628.17$$

(*) video here.

Euler's number, e .

- $e \sim 2.71828$

- $e = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots$

- $e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$

The exponential function, $f(x) = e^x$.

The natural logarithm, $\log_e x = \ln x$.