Warm up. $\log_3(x+5)+\log_3(x-1)=2$

$\log_3[(x+5)(x-1)]=2$

$3^2=(x+5)(x-1)$

$9=x^2+4x-5$

$0=x^2+4x-14$ (*) use the quadratic formula

$x=\frac{-4\pm\sqrt{16+56}}{2}=-2\pm3\sqrt{2}$ // check the validity of these solutions

$-2+3\sqrt{2}-1=-3+3\sqrt{2}>0$

$-2+3\sqrt{2}+5=3+3\sqrt{2}>0$

$-2-3\sqrt{2}-1=-3-3\sqrt{2}<0$ NOT VALID

$x=-2+3\sqrt{2}$ is the only solution

Logarithm of a quotient. (our last property)

$$\log_b\left(\frac{x}{y}\right) = \log_b(x \cdot y^{-1})$$

log of a product

$$= \log_b(x) + \log_b(y^{-1})$$

log of an exponent

$$= \log_b(x) - \log_b(y)$$

$$\boxed{\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)}$$

Ex. $\log_2(x+3) - \log_2(x+1) = 1$

$$\log_2\left(\frac{x+3}{x+1}\right) = 1$$

$$\frac{x+3}{x+1} = 2^1 = 2$$

$$x+3 = 2(x+1)$$

$$x+3 = 2x+2$$

$$1 = x$$ (*) Check its valid! It is!

Logarithm properties worksheet
Math 32 Spring 2022 Name: _____

1. Solve for x.
   (a) $\log_5 25 = x$
   (b) $\log_3 \frac{1}{81} = x$
   (c) $\log_2 x = -3$
   (d) $\log_9 x = -\frac{1}{2}$
   (e) $\log_x 64 = 3$

(a) $x = 2$ (b) $x = -4$
(c) $x = \frac{1}{8}$ (d) $x = \frac{1}{3}$
(e) $x = 4$

2. Write the following expressions in terms of $\log x$, $\log y$, and $\log z$.
   (a) $\log\left(\frac{xy^3}{z}\right)$
   (b) $\log\left(x\sqrt[3]{xy}\right)$
   (c) $\log\left(z^3\sqrt{\frac{x}{\sqrt{y}}}\right)$

(a) $\log x + 3\log y - \log z$
(b) $\frac{4}{3}\log x + \frac{1}{3}\log y$
(c) $3\log z + \frac{1}{2}\log x - \frac{1}{4}\log y$

3. Solve the following equations.
   (a) $2\log x = \log 2 + \log(3x - 4)$
   (b) $\log_5 x + \log_5(x - 1) = \log_5(4x)$
   (c) $\log_3(x + 25) - \log_3(x - 1) = 3$
   (d) $\log_2(x - 2) + \log_2(x + 1) = 2$

(a) $x = 2, 4$
(b) $x = 5$
(c) $x = 2$
(d) $x = 3$