3.2 & 3.3 Properties of Logarithms.

Warm up. Solve for x, $2^{2x} = 7$.

① $2x = \log_2 7$

$x = \frac{1}{2}\log_2 7$

② $2^x = 7^{1/2}$

$x = \log_2 7^{1/2}$

③ $4^x = 7$

$x = \log_4 7$

Goal of today: understand why all the above expressions are equivalent.

Property 1: log of a power

$b>0, y>0, b\neq 1$ then $\boxed{\log_b y^t = t \cdot \log_b y}$

Let's see an example.

$\log_2 4^3 = \log_2 64 = 6$

$\Updownarrow$

$2^x = 64$

this is easier bc numbers stay smaller

$\log_2 4^3 = 3\log_2 4 = 3 \cdot 2 = 6$

Now we can show ① = ②,

$\log_2 7^{1/2} = \frac{1}{2}\log_2 7$

Ex. Suppose $\log_3 t = 2.48$, compute $\log_3(t^{100})$.

$\log_3(t^{100}) = 100 \log_3 t = 100 \cdot 2.48$

$= 248$

Property 2: change of base for logs.

$a, b, y > 0$ and $a \neq 1, b \neq 1$, then $\boxed{\log_b y = \frac{\log_a y}{\log_a b}}$

Ex Write $x = \log_9 17$ as a log with base 3.

$y = 17$, $b = 9$, $a = 3$ ← the base I want

$\log_9 17 = \frac{\log_3 17}{\log_3 9} = \frac{\log_3 17}{\log_3 3^2} = \frac{\log_3 17}{2\log_3 3} = \frac{\log_3 17}{2}$

$\Updownarrow$

$\frac{1}{2}\log_3 17$

Let's see a little proof of this property,

Suppose $x = \log_b y$ (*)

$$\Leftrightarrow b^x = y$$

$$\Leftrightarrow \log_a b^x = \log_a y$$

$$\Leftrightarrow x \log_a b = \log_a y$$

$$\Leftrightarrow x = \frac{\log_a y}{\log_a b} \quad (**)$$

Equating (*) and (**) gives, $\log_b y = \frac{\log_a y}{\log_a b}$. □

Now we see that ③ = ① (and also = ②),

$$x = \log_4 7 = \frac{\log_2 7}{\log_2 4} = \frac{\log_2 7}{2} = \frac{1}{2}\log_2 7.$$

Suppose $x = \log_b s$ & $y = \log_b t$,

$\Updownarrow$ $\Updownarrow$

$b^x = s$ $b^y = t$

let's multiply these two equations

$\Rightarrow b^x b^y = st$

$\Leftrightarrow b^{x+y} = st$

$\Leftrightarrow x + y = \log_b(st)$

$\Leftrightarrow \log_b(s) + \log_b(t) = \log_b(st)$

Property 3: log of a product.

$b, x, y > 0$ & $b \neq 1$ $\boxed{\log_b(xy) = \log_b x + \log_b y}$

EX. Let $\log_3 x = 4.5$, $\log_3 y = 3.1$

Compute $\log_3(12xy)$,

$\log_3(12xy) = \log_3 12 + \log_3(xy)$

$= \log_3 4 \cdot 3 + \log_3 x + \log_3 y$

$= \log_3 4 + \log_3 3 + 4.5 + 3.1 = \log_3 4 + 8.6$