3.1 Logarithms

Warm up. Suppose $3^x = 27$. Solve for x.

$3^x = 27$
$3^x = 3^3$
$x = 3$

So what happens if there is no like base?

$3^x = 18$

There is no obvious way to write 18 as a power of 3.

Use the **logarithm** as the inverse of exponentiation.
The above expression is equivalent to,

$x = \log_3 18$

log base 3 of 18

$$\boxed{\log_b y = x \iff b^x = y}$$

Ex. $\log_2 4 = ?$

$\log_2 4 = x \iff 2^x = 4$

$x = 2$

Ex. $\log_3 \frac{1}{27} = ?$

$\log_3 \frac{1}{27} = x \iff 3^x = \frac{1}{27}$

$x = -3$

Ex. Find x such that $2^{x+3} = 7$

$2^{x+3} = 7 \iff \log_2 7 = x+3$

$x = \log_2 7 - 3$

Basic properties of logs.

① $\log_b 1 = 0$.

② $\log_b b = 1$.

Exercise. Confirm the above by translating them into expressions about exponents.

Logarithm as the inverse of exponential.

$f(x) = b^x$, $b > 0$ & $b \neq 1$

$f^{-1}(x) = \log_b x$

Note: when the book writes $\log x$ without indicating a base, they mean $\log_{10} x$.