2.3 (sort of) Exponents.

Goal: Feel comfortable manipulating expressions w/ exponents.

Let m be a positive integer...

$$x^m = \underbrace{x \cdot x \cdots x}_{m \text{ times}}$$

Some immediate consequences of this,

$$x^m x^n = x^{m+n}$$
$$(x^n)^m = x^{nm}$$
$$x^n y^n = (xy)^n$$

$n, m \in \mathbb{N}$

"natural numbers"

<u>Ex.</u> Simplify x^3xy^4

$$x^3 x y^4 = x^3 x^1 y^4 = x^4 y^4 = (xy)^4$$

$x = x^1$

Now let's define x^0...

$$x^n = x^{n+0} = x^n x^0$$
$$x^n = x^n x^0$$
$$\boxed{1 = x^0}$$

The book claims (poorly IMO) the 0^0 is undefined. I do not believe this, I believe $0^0 = 1$.

Now let's define x^{-m} ...

$$x^m x^{-m} = x^{m-m} = x^0$$
$$x^m x^{-m} = 1$$
$$\boxed{x^{-m} = \frac{1}{x^m}}$$

similarly, $\frac{1}{x^{-m}} = x^m$

__Ex.__ Simplify $\frac{x^{-5}}{x^2}$

$$\frac{x^{-5}}{x^2} = \frac{1}{x^5 x^2} = \frac{1}{x^7}$$

__Ex.__ Simplify $\frac{x^3}{x^6}$

(*) $\frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}$

$$\frac{x^3}{x^6} = \frac{1}{x^{-3} x^6} = \frac{1}{x^3}$$

Expressing the **square root** as an exponent,

$$\sqrt{x} = x^{1/2}$$

Where the square root is the number such that

$$\sqrt{x}\sqrt{x} = x \quad (\text{for } x \geq 0)$$

We can also define the **m-th root** similarly,

$$\underbrace{\sqrt[m]{x}\,\sqrt[m]{x}\cdots\sqrt[m]{x}}_{m\text{-times}} = x$$

For example,

$$\sqrt[3]{8} = 2, \quad 2 \cdot 2 \cdot 2 = 8$$

We can also express the m-th root in terms of an exponent,

$$\sqrt[m]{x} = x^{1/m}$$

__Ex.__ Simplify $\dfrac{(x^{-2})^3 y^8}{x^{-5}(y^4)^{-3}}$

$$\frac{x^{-6}y^{8}}{x^{-5}y^{-12}} = x^{-6+5}y^{8+12} = x^{-1}y^{20}$$
$$= \frac{y^{20}}{x}$$

__Ex.__ Simplify $\dfrac{x^3(y^2x)^2}{x^{-5}y^2y^{-4}}$

$$= \frac{x^3y^4x^2}{x^{-5}y^{-2}} = x^{3+2+5}y^{4+2} = x^{10}y^6$$