An overview of midterm topics:

1. Absolute value & inequalities ✓
2. Domain & range of functions ✓✓
3. Function transformations ✓
4. Inverses ✓
5. Composition
6. Lines ✓
7. Quadratics / conics ✓
8. Polynomials ✓

Now for some practice problems:

1 (A) Compute the slope-intercept form of the line that is perpendicular to the line $y = 3 - x$ and passes through the point $(3,5)$.

perpendicular $\Rightarrow$ $m = 1$

$y = x + b$

$5 = 3 + b \Rightarrow b = 2$

$y = x + 2$

(B) Compute where these two lines intersect.

$x+2=3-x$

$2x=1$

$x=\frac{1}{2}$

$y=\frac{1}{2}+2$

$y=\frac{5}{2}$

$\left(\frac{1}{2}, \frac{5}{2}\right)$

② Find all x that satisfy $|x|+|3x-1|=17$.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases} \qquad |3x-1| = \begin{cases} 3x-1 & x \geq \frac{1}{3} \\ -3x+1 & x \leq \frac{1}{3} \end{cases}$$

(i) $x \leq 0 \Rightarrow -x-3x+1=17$

$-4x=16$

$x=-4$

(ii) $0 \leq x \leq \frac{1}{3} \Rightarrow x-3x+1=17$

$-2x=16$

$x=-8$ not in domain

(iii) $x \geq \frac{1}{3} \Rightarrow x+3x-1=17$

$4x=18$

$x = \frac{9}{2}$

③ (A) Compute the inverse of $f(x) = \frac{2x-1}{x+4}$

$$y = \frac{2x-1}{x+4}$$

$$y(x+4) = 2x-1$$

$$yx+4y-2x = -1$$

$$x(y-2) = -4y-1$$

$$x = \frac{-4y-1}{y-2} = f^{-1}(y)$$

(B) What is the domain & range of f^{-1}?

$$f^{-1}(y) = \frac{-4y-1}{y-2} \Rightarrow y \neq 2$$

domain is all reals except 2.

range f^{-1} = domain f

$f(x) = \frac{2x-1}{x+4} \Rightarrow x \neq -4 \Rightarrow$ range of f^{-1} is all reals except -4.

④ Find the center of the circle with equation

$$x^2 - 6x + y^2 + 10y = 1$$

$$x^2 - 6x = (x-3)^2 - 9$$

$$y^2 + 10y = (y+5)^2 - 25$$

$$(x-3)^2 - 9 + (y+5)^2 - 25 = 1$$

$$(x-3)^2 + (y+5)^2 = 35$$

Center @ $(3, -5)$

⑤ Suppose $f(x)$ is a function defined on the domain $[0,3]$ with range $[-1,4]$. Let $g(x) = 2f(x+1) - 1$. What is the domain and range of $g(x)$.

Horizontal changes change domain:

- shift LEFT one

$[0,3] \rightarrow [-1,2]$ domain of g

Vertical changes change range:

- stretch by 2
- shift up one

$[-1, 4] \to [-2, 8] \to [-1, 9]$ range of g

© Given an example of a polynomial with integer coefficients such that $\sqrt{3}$ is zero.

$p(x) = (x - \sqrt{3})$ ← not an integer so we need to multiply by something

$p(x) = (x - \sqrt{3})(x + \sqrt{3})$

Check coef by distributing,

$$(x - \sqrt{3})(x + \sqrt{3}) = x^2 - \sqrt{3}x + \sqrt{3}x - 3$$
$$= x^2 - 3 \quad \checkmark$$