A **rational function** is a function r s.t.

$$r(x) = \frac{p(x)}{q(x)}$$

where p and q are polynomials & $q \neq 0$.

Exercise. Compute the domain of the rational function,

$$r(x) = \frac{x+1}{x^2-4}$$

The denominator cannot equal 0 !!

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

Let's talk about **asymptotes**.

An **asymptote** is where the graph becomes and stays arbitrarily close to the line in at least one direction.

(1) Vertical asymptotes – occure when the denominator equals 0.

(2) Horizontal asymptotes — occur when the function tends to a constant as $x \to \pm\infty$

Behavior of $r(x)$ as $x \to \pm\infty$.

Separate the highest degree term of the numerator & denominator.

Ex. $r(x) = \dfrac{x^2+x-3}{2x+1}$

as $|x| \to \infty$, $r(x) \sim \dfrac{x^2}{2x} = \dfrac{1}{2}x$

since this is not a constant (depends on x) we do not refer to this as a asymptote.

Ex. $r(x) = \dfrac{3x^3+x}{-x^3+x+2}$

As $|x| \to \infty$, $r(x) \sim \dfrac{3x^3}{-x^3} = -3$

horizontal asymptote.

(*) demos.